

# Interplay Between Traffic Dynamics and Network Structure

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**Abstract**—This paper studies the interplay between traffic dynamics and network structure in complex communication networks. Complex communication networks of distinct structural features are chosen as the underlying networks. We use node betweenness centrality, network polarization, and average path length to capture the structural characteristics of a network. Network throughput and average packet delay are the main performance measures. We study how internal traffic, throughput, and delay change with increasing incoming traffic through simulation. We further investigate the relationship between network performance and network structure. Our work reveals that the parameters chosen to reflect network structure, including node betweenness centrality, network polarization, and average path length, play important roles in different states of the underlying networks.

**Keywords**—Complex networks; traffic; network structure; network performance.

## I. INTRODUCTION

Many social, biological, and communication systems are called complex systems. In network science, complex systems are described as networks consisting of vertices and interactions or connections among them. The study of structural and dynamical properties of complex systems has been receiving a lot of interests. One of the ultimate goals of the studies is to understand the influence of topological structures on the behaviors of various complex systems, for instance, how the structure of social networks affects the spread of diseases, information, rumors, or other things [1-3]; how the structure of a food web affects population dynamics [4-5]; how the structure of a communication network affects its robustness, reliability [6-7], and so on.

There is a wealth of literature focusing on different performance aspects of communication networks. By viewing communication networks as weighted graphs, authors in [7-9] have developed a concept called network criticality. They found that network criticality directly relates to network performance metrics such as average network utilization and average network cost. Most network centrality indices have structural significance. In [10], the authors compare different centrality indices for the measuring of nodal contribution to global network robustness. Since the discovery of power-law degree distribution of the Internet topology [11], much effort has been made on the study of scale-free (SF) networks. In

[12-17], different routing strategies have been proposed in order to improve the performance of SF networks. To enhance the traffic transport efficiency of SF networks, an optimal resource allocation scheme is presented in [18]. Lattice networks are widely used, for example, in distributed parallel computation [19], distributed control [20], satellite constellations [21], and sensor networks [22]. Authors in [22] study the effect of routing on the queue distribution, and investigate the routing algorithms in lattice networks that achieve the maximum rate per node under different communication models.

In our previous work [23], we compared the latency of SF networks and random networks under different routing strategies. In order to better understand the structural influence on the performance of communication networks, in this paper, we devote ourselves to explore the relationship between network structure and network performance under dynamic input traffic. Four different types of networks are chosen as the underlying networks. They are SF networks, square lattice (SL) networks, random networks, and ring lattice (RL) networks. We use node betweenness centrality, network polarization, and average path length, to capture the structural features of different networks. Since both throughput and delay are especially important for communication networks, they are used here as main performance measures.

In the work, based on observed traffic dynamics in the networks studied, three network states are classified: traffic free flow state, moderate congestion state, and heavy congestion state. Simulation results indicate that during each different state, the structural differences among the underlying networks play important roles in network performance. Through the work, it is possible that a better comprehension of the interplay between traffic dynamics and network structure could help in designing better network structures and better routing protocols.

The paper is organized as follows. Section II presents our network model. Simulation results and analysis are provided in Section III. Section IV concludes the work.

## II. NETWORK MODEL

In the paper, four different types of networks are chosen as the underlying networks. They are the SF network, the random network, the SL network, and the RL network. One of their

structural differences lies in their distinct nodal degree distribution. The degree of a node is the total number of links connecting it. The SF network is built based on the Barabasi-Albert (BA) model proposed in [24]. It has a power law degree distribution so that most nodes have very low degrees, but a few nodes (called hubs) could have extremely high degrees. The random network is formed according to the Erdős-Rényi (ER) model proposed in [25]. The random ER network follows Poisson degree distribution when network size is large. In the random ER network, the degrees of most nodes are around the mean degree. In the SL network, all the nodes except those located on the edge of the square have the same degree. The RL network is constructed by connecting each node on a circle to its  $2m$  ( $m \geq 1$ ) nearest neighboring nodes. Apparently, all the nodes in the RL network have the same degree.

In the paper, we use node betweenness centrality, network polarization, and average path length to capture the structural characteristics of complex communication networks. The node betweenness  $B_i$  for a node  $i$  is defined here as the total number of shortest path routes passing through that node. Nodes with high betweenness values participate in a large number of shortest paths. Therefore, initial congestion usually happens at nodes of the highest betweenness value. Node betweenness reflects the role of a node in a communication network. Normally, high betweenness nodes also have high degrees. The node betweenness distribution of a communication network is demonstrated through a measure of the polarization,  $\pi$ , of the network [26]. It is defined as:

$$\pi = \frac{B_{max} - \langle B \rangle}{\langle B \rangle} \quad (1)$$

Where  $B_{max}$  is the maximum betweenness value,  $\langle B \rangle$  is the average betweenness value. We find that  $\pi$  as an indication of node betweenness distribution suits our work better than others (e.g. standard deviation). The large polarization value of a network tells us that at least one node possesses much larger betweenness values than most of the other nodes in the network. Therefore, the larger the value  $\pi$  is, the more heterogeneous the network is. On the other hand, for very homogeneous networks,  $\pi$  is very small. For example, for the RL network, we have  $\pi \approx 0$ . The average path length  $\langle D \rangle$  of a network is defined as the average of the shortest path lengths among all the source-destination pairs. We will show in the next section that the average path length directly relates to the total amount of internal traffic in the network. It also relates to average packet delay.

The above three parameters capture the structural features of a network from different angles. They are also interrelated. Usually, the more heterogeneous (larger  $\pi$ , or relatively higher  $B_{max}$ ) the network is, the shorter the average path length  $\langle D \rangle$  is. The reason is that high betweenness (or degree) nodes serve as shortcuts for connecting node pairs. In addition, the following relationship between shortest path length and node betweenness centrality can be easily found,

$$\sum_{i,j} D_{ij} = \sum_i B_i \quad (2)$$

Where  $D_{ij}$  stands for the shortest path length from node  $i$  to node  $j$ ,  $B_i$  stands for the betweenness value of node  $i$ .

The networks are treated as packet-switched networks. In these networks, fixed shortest path routing strategy is implemented. The length of the shortest path is the minimum hop count between a source-destination pair. Given network topology, each node calculates the shortest paths to all the other nodes using Dijkstra's algorithm. Then a routing table is constructed at each node. A routing table contains three columns: the destination node, next node to route a packet to the destination, and the hop count to the destination.

In the networks studied, traffic dynamic is governed by the following network model, similar to the one discussed in [27]. In the model, we assume that time is slotted. During each time slot, first, packets are generated at each node  $i$  with a rate  $\lambda_i$ , the destination of a packet is randomly chosen among all other nodes. Each node is endowed with a first-in-first-out (FIFO) queue in which packets are stored waiting to be processed. Then, if its queue is not empty, each node  $i$  transmits packets at a rate  $r_i$ , which represents bandwidth, to one of its neighbors according to its routing table. When a packet reaches its destination, it is absorbed by the destination node. For simplicity, for all the nodes, we assume the packet generation rate is the same or let  $\lambda_i$  equals to  $\lambda$ . We also assume  $r_i$  equals to 1, which means during each time slot, each node can process one packet.

We use throughput and average packet delay as two main performance measures. Throughput is defined as the average number of delivered packets per time slot. The average packet delay is defined as the average time that a delivered packet spent in the network. Our task is by observing traffic dynamics in different networks, to find out the relationship between network structure and network performance.

### III. SIMULATION RESULTS AND ANALYSIS

In the simulation, a discrete time clock  $k$  is used. Simulation starts with  $k = 0$ , for each passed time slot,  $k$  is incremented by 1. The performance of a packet-switched network is measured by its throughput  $o(k)$  and average packet delay  $\tau(k)$ .

TABLE I  
NETWORK PARAMETERS

	$B_{max}$	$\langle B \rangle$	$\pi$	$\langle D \rangle$
SF network	802	127	5.32	2.59
Random ER	449	141	2.19	2.87
SL network	376	224	0.68	4.67
Ring lattice	416	325	0.28	6.63

Both  $o(k)$  and  $\tau(k)$  are calculated respectively as the average from the start of simulation ( $k = 0$ ) to time  $k$ . We use  $n(k)$  to represent the total number of packets within the network at time  $k$ . In the simulation, the SF network, the random ER

network, and the RL network are all generated with 50 nodes and 100 links. The SL network is generated with 49 nodes and 84 links because of its structural restriction. Simulation includes two parts. The first part investigates how  $n(k)$  change as a function of  $\lambda$  and  $k$ . In this part, we observe a network phase transition from traffic free flow to congestion as reported in [27-28]. The second part investigates network performance as a function of  $\lambda$ . Three network states are classified accordingly. At last, we demonstrate that how, in different network states, the structure of a network influences its performance.

Table I lists the related parameters of the underlying networks. It tells us that the RL network has the longest average path length  $\langle D \rangle$ ; while the average path length of the SF network is the shortest. In addition, the RL network has the lowest polarization value  $\pi$ , which shows its almost homogeneous structure in terms of node betweenness distribution; while the SF network has the highest  $\pi$ , which demonstrates its most heterogeneous structure. The corresponding parameters of the random ER network and the SL network lie somewhere in between. One exception is that the SL network has the lowest  $B_{max}$ . In our simulation, each data obtained is averaged over 100 runs.

A.  $n(k)$  vs.  $\lambda, k$

This part investigates the change of  $n(k)$  as a function of  $\lambda$  and  $k$  in the networks studied. Simulation results are plotted in Fig.1 and Fig. 2.

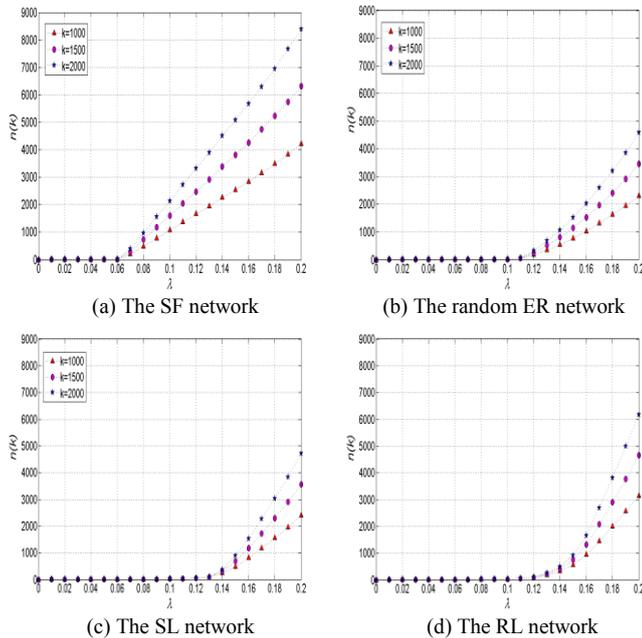


Fig. 1  $n(k)$  as a function of  $\lambda$  ( $k = 1000, 1500, 2000$ )

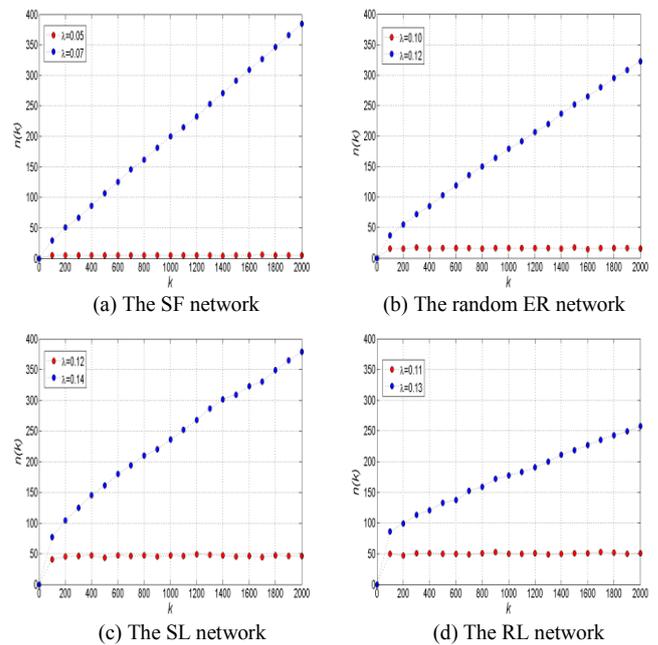


Fig. 2  $n(k)$  as a function of  $k$  for subcritical and supercritical values of  $\lambda$

Fig. 1 shows that all four networks display similar performance. When the incoming traffic  $\lambda$  increases, a critical point  $\lambda_c$  is observed in all these networks where a network phase transition takes place from traffic free flow to congestion. Fig. 2 presents the change of  $n(k)$  as a function of time  $k$  for subcritical and supercritical values of  $\lambda$ . In the case of subcritical value of  $\lambda$ ,  $n(k)$  remains constant; while in the case of supercritical value of  $\lambda$ , we observe continuous accumulation of packets in the networks with the passing of time  $k$ .

When  $\lambda < \lambda_c$ , a network is in steady state or traffic free flow state. In this state,  $n(k)$  remains almost unchanged with the increase in incoming traffic  $\lambda$ , and/or time  $k$ . However, for different networks,  $n(k)$  is proportional to the average path length of a network (shown in Fig. 2). According to Little's law, for a network of size  $N$ , the number of packets created per unit time (given by  $N \times \lambda$ ) must be equal to the number of packets delivered per time slot. Since the number of delivered packets per time slot is  $\frac{n(k)}{\tau(k)}$ , hence  $\frac{n(k)}{\tau(k)} = N\lambda$ .

When  $\lambda > \lambda_c$ , the networks enter into congestion state, where  $n(k)$  start increasing quickly with the increase in  $\lambda$ , and /or time  $k$ . From Fig. 1, we observe that compared with the other networks, the SF network has the lowest value of  $\lambda_c$ . The reason lies in its highest  $B_{max}$  among all the networks studied. According to the definition of node betweenness, the node with maximum betweenness value  $B_{max}$  has to handle the heaviest traffic because it participates in the largest number of shortest path routes. With increasing incoming traffic, initial congestion (or quick accumulation of packets) shall take place first at the node with  $B_{max}$ . The results conform to the theoretical analysis provided in [17].

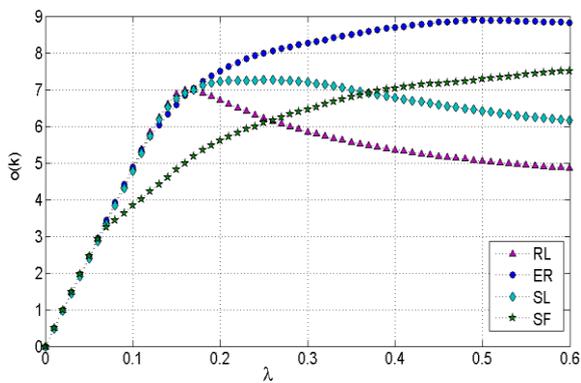
B.  $o(k), \tau(k)$  vs.  $\lambda$

Performance comparison among the networks is shown in Fig. 3 in terms of throughput  $o(k)$  and average packet delay  $\tau(k)$ . Based on network performance, three network states are classified: traffic free flow state, moderate congestion state, and heavy congestion state.

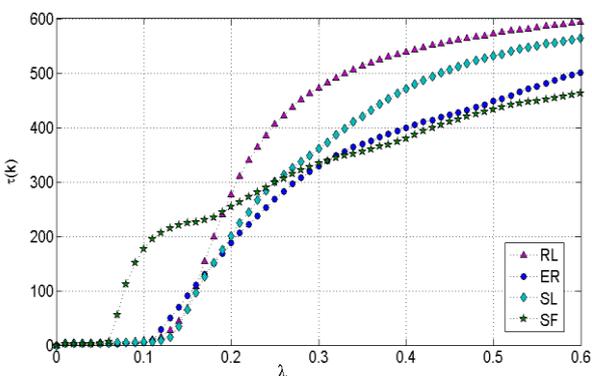
As stated in previous section, when  $\lambda < \lambda_c$ , a network is said to be in traffic free flow state. From Fig. 3 (a), we observe that in this state, all the networks perform the same in terms of throughput (throughput increases linearly with  $\lambda$ ), but not so in terms of average packet delay. In traffic free flow state, from  $\frac{n(k)}{\tau(k)} = N\lambda$ , we obtain  $\tau(k) = \frac{n(k)}{N\lambda}$ . Since  $n(k)$  depends on

the average path length of a network, the average packet delay  $\tau(k)$  also depends on the average path length  $\langle D \rangle$  of the network. Our simulation results show that the SF network has the lowest  $\tau(k)$  because it has the shortest average path length. Therefore, in traffic free flow state, the average path length plays a major role in network performance.

When  $\lambda$  exceeds the critical point  $\lambda_c$ , congestion happens because packets start to accumulate in the network. When  $\lambda > \lambda_c$ , Fig. 3 (a) shows that with continuous increase in  $\lambda$ , the increase in throughput becomes slower. We say that a network is in moderate congestion state. With further increase in  $\lambda$ , if the throughput starts to decrease, we say that the network has entered into heavy congestion state.



(a)  $o(k)$  vs.  $\lambda$



(b)  $\tau(k)$  vs.  $\lambda$

Fig. 3 Performance comparison among the networks ( $k = 2000$ )

From Fig. 3, we find that the SF network is the first that enters into moderate congestion state, during which the increase in throughput slows down, and its average packet delay starts to increase very quickly. Compared to the others, the performance of the SF network is the worst. The reason lies in its most heterogeneous structure (largest  $\pi$ ). In the SF network, huge amount of packets start to accumulate at one or several nodes of extremely high betweenness values when many other nodes are idle (or do not have enough packets to send). A similar phenomenon is observed in the random ER network in moderate congestion state, but the random ER network performs much better than the scale-free network because of its much smaller polarization value  $\pi$ . According to the same reasoning, we find that both the RL network and the SL network achieve higher throughput and lower delay than the other two because of their much lower polarization value  $\pi$ . However, Fig. 3 shows that with just a little increase in  $\lambda$ , they quickly enter into heavy congestion state. We find that even though in moderate congestion state, congestion happens at only a few nodes, the performance of a network depends heavily on the traffic load distribution. The less the value of network polarization is, the more homogeneous (in terms of node betweenness distribution) a network is, the more balanced the traffic load is distributed; therefore, the better the network performs. For the RL and SL networks, their almost uniform node betweenness distribution results in more balanced traffic load distribution among all the nodes so that many packets are delivered successfully. Therefore, we may say that in moderate congestion state when traffic is not yet very heavy, network performance strongly relates to network polarization.

When  $\lambda$  increases beyond a specific value (this value is different for different networks), the networks enter into heavy congestion state. In this state, network throughput starts to decrease. For the SF network and the random network, because of their heterogeneous structure (large  $\pi$ ), most traffic is jammed at more nodes of high betweenness values, only a small amount of traffic bypassing those congested nodes can still be delivered successfully. However, compared to the SF network, the performance of the random network is much better because the random network is relatively less heterogeneous (relatively smaller  $\pi$ ). For the RL network and the SL network, their structures are more homogeneous. However, since the incoming traffic becomes very heavy, their very long average path length and high average betweenness value causes huge amount of internal traffic. In addition, since their node betweenness distribution is almost uniform, almost all the nodes are congested (few packets can be delivered successfully). Compared to the RL network, the SL network performs better because of its relatively shorter average path length and lower betweenness values. Therefore, in heavy congestion state, both average path length and node betweenness distribution play important roles in network performance.

The above analysis is verified by our observation on the changes in queue length (total number of packets in a queue) through simulation. In traffic free flow state (we choose  $\lambda =$

0.05), most queues in all the networks are almost empty. In moderate congestion state (we choose  $\lambda = 0.13$ ), most queues in the RL network contain several packets, a few queues contain several tens of packets, and the length of one queue exceeds one hundred packets. It is similar for the SL network. Most queues in the random network are almost empty, but the queues at a few nodes of high betweenness values contain hundreds of packets. Similar to the random network, most queues in the SF network are almost empty, but two queues at two nodes of extremely high betweenness values contain thousands of packets respectively. In heavy congestion state (a different  $\lambda$  is chosen for each network), for the RL network, the whole network is congested (most queues contain several tens of packets, a few queues contain even hundreds of packets). It is similar to the SL network. While for the random network and the SF network, more than half of the queues are still almost empty, more nodes of high betweenness values are heavily congested. Interestingly, we find that no matter what the structure of the underlying network is, congestion always takes place when a large number of packets start to accumulate at a few nodes.

#### IV. CONCLUSIONS

We have investigated how internal traffic, throughput, and average packet delay change as a function of incoming traffic in networks of different structures. Three network states have been classified: traffic free flow state, moderate congestion state, and heavy congestion state. Network performance has been measured and compared in terms of throughput and average packet delay. Under fixed shortest path routing, node betweenness, network polarization, and average path length all play important roles in different states of the underlying networks. In traffic free flow state, average path length plays the major role; it directly affects average packet delay. In moderate congestion state and heavy congestion state, both average path length and node betweenness distribution play important roles in network performance. Based on our investigation, an optimal network structure should have short average path length (which results in less total internal traffic), and small network polarization  $\pi$  (which leads to more balanced traffic load distribution).

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