Local flow distribution and optimal weighting against congestion and cascading failures in weighted complex networks

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Abstract-Network robustness against congestion and cascading failures has been a research topic of practical importance. In this paper, we study information flow in weighted complex networks. We use a simple model to simulate traffic dynamics in the network, coupled with a local flow distribution strategy. Such flow distribution only makes use of the knowledge of local link weights. By implementing the model in the underlying scale-free (SF) networks, we demonstrate that by properly tuning a weight parameter, it is possible to achieve a global flow optimization based on local flow distribution, and a maximal level of network robustness against congestion and cascading failures can be obtained simultaneously. Numerical evidences have been provided in supporting our findings.

1. Introduction

A complex network typically consists of a large number of nodes or components interacting with each other in a complicated manner. In such a system, small, local failures may trigger system breakdown on a global scale through mechanism of cascading. In particular, heavy congestion and cascading failures are critical issues for a large number of technological networks including power grids, information communication networks, and transportation networks. Typical examples include electric power grid blackouts and the Internet collapse. For instance, the Western North America blackouts took place in 1996; the largest blackouts in US history took place in 2003; and the Internet services disruption took place in December 2006 in various parts of Asia because of the broken of several undersea cables caused by the submarine earthquake near Tai-wan [1]. Therefore, research on the control and defense of congestion and cascading failures is of practical importance.

By adopting different local load redistribution rules, authors in [1-5] have investigated, respectively, the process of cascades after different types of node/link failures occur, and obtained the estimate for cascading size. Systemic failure cascade has been investigated in a regular network with load redistributed equally to the neighbors of the failed node [6]. A. E. Motter [7] has proposed a strategy based on selective removal of nodes and edges, which is shown to drastically reduce cascading size. A matching model of capacity [8] has been proposed to defense cascading failures triggered by the removal of the node with maximum load.

In this paper, we investigate how to enhance network robustness for the defense of heavy congestion and cascading breakdown. Authors in [9] have explored flow optimization and cascading effects in complex networks by studying current flow in resistor networks, where flow is directed along all possible paths between source and destination node. A similar work has been done in [10], where traffic flow is directed to follow the weighted shortest path between source and destination node. In fact, as global flow distribution strategy, flow distribution based on the shortest path method has been widely used in many communication networks of small and medium sizes due to its simplicity and low cost. However, for large scale complex networks, it is not practical because of its high communication cost in search the network in real time and other related issues [11].

In our work, a local flow distribution strategy is implemented and studied in weighted complex networks. The probability of a flow being distributed from one node to one of its neighboring nodes solely depends on local information of link weights. In this way, not only no global information is needed, but also flow distribution would not be affected much even when topological changes take place in some parts of the network. Our study focuses on scale-free

(SF) networks because many real-world networks have been proven to have scale-free structure including power networks [12-13], and the Internet [14].

The main contribution of this paper is that we implement a random walk-based local flow distribution strategy, coupled with a link weighting scheme in weighted complex networks; through computations and simulations, we demonstrate that by properly tuning a weight parameter θ, a global flow optimization can be achieved based on local flow distribution; more importantly, network robustness against congestion and cascading failures reaches maximal simultaneously. Our work aims at reducing the probability of congestion and cascades to a minimal level, different from the purposes of the random walk approach employed in [15-16]. We use a simple model to simulate the dynamics of traffic flow in the network. The network traffic capacity is characterized as the maximum packet generation rate at which the network is free of congestion. The higher the maximum packet generation rate, the more the traffic that the network can handle. We study the severity of congestion and cascades for different weight parameter θ. Under an optimal link weighting, a global traffic optimization is observed, and network robustness is found to reach maximal. Enhanced network robustness is demonstrated particularly through the minimization of the maximum nodal load, and the maximization of network traffic capacity achieved simultaneously. Compared with the values realized in the corresponding nonweighted networks, the improvement is quite striking. In a practical sense, our work actually shows that a network can essentially be made congestion- and cascade- free.

This paper is organized as follows. In Section 2, we introduce the random walk-based local flow distribution strategy, and a link weighting scheme. Section 3 provides the computational and simulation results. Conclusions are drawn in Section 4.

2. Random Walk-based Local Flow Distribution and Link Weighting Scheme

Our work is inspired by A. Tizghadam's view on node betweenness centrality based on random walk, which is obtained by following the method introduced in [17]. A network is treated as a weighted connected graph G(N, E, W), where N is the set of vertices (or nodes), E is the set of edges (or links), and W is the set of link weights. Let us assume a random walk from source node s to destination node d. The probability of this random walk passing through node i in next step is represented by $p_{si}(d)$ and is defined as,

$$p_{si}(d) = \frac{w_{si}}{\sum_{(s,q)\in E} w_{sq}}(1-\delta_{sd}) \qquad (1)$$

Where w_{si} is the weight of link (s, i) (if there is no link between s and i, then $w_{si}=0$), and δ_{sd} is the Kronecker delta function (if s = d, $\delta_{sd}=1$; otherwise, $\delta_{sd}=0$). A random walk terminates exactly with the occurrence of the first arrival to the destination node d.

The random walk betweenness B_i of node i represents the total expected number of times a random walk traverses node i over all source-destination pairs. It is given by,

$$B_i = \frac{1}{2} W_i R_{tot} \qquad (2)$$

Where $W_i = \sum_{j\in A(i)} w_{ij}$, is the weight of node i, $A(i)$ is the set of adjacent nodes of node i. R_{tot} is called the total resistance distance [18]. The value of R_{tot} depends on network topology and link weights. If the network is viewed as a resistor network, where the conductance of a link equals the weight of the corresponding link in original network, the effective resistance R_{ij} between a pair of nodes i and j is the electrical resistance measured across nodes i and j. R_{tot} is the sum of the effective resistances between all distinct pairs of nodes. Thus, it is a global parameter. R_{tot} is proven closely relate to some important performance metrics of communication networks [19]. It is a monotone decreasing function of link weights [17-19].

Assume that for each distinct source-destination pair in the network, unit information is generated at the source node, and delivered to the destination node. If the probability of the information distributed to a node i in next step is given by Eq. (1), then the random walk betweenness of a node captures appropriately the amount of information passing through it. We refer to it as the information load of the node (or nodal load). For traffic flow dynamics, initial congestion takes place when traffic starts to quickly accumulate at a node, which causes the overload, and probable failure of the node (depending on its sensitivity to overload). Cascading failures take place when load redistribution of a failed node causes an avalanche of the failures of other nodes. It may cause even more damage to the network when the node with maximum load fails [5, 8]. Congestion and cascading failures thus closely relate to each other. An overloaded node may fail because of its limited processing power (or capacity). Assume that all the nodes have the same processing power, and all the links have sufficient capacities, the congestion- and cascade- free state of the network is mainly limited by the maximum nodal load, which is given by,

$$B_{\max} = \max\{B_i, i = 1,2,...,n\} = \frac{1}{2} W_{\max} R_{tot} \qquad (3)$$

Let us assume that the weight of an arbitrary link $(i, j) \in E$ is set to be proportional to the end-node degrees, namely, $w_{ij} = (k_i k_j)^{\theta}$, k_i is the degree of node *i*, θ is a control parameter, taking only real values. This weighting choice, also studied in [3, 9-10], has been motivated by empirical studies [20-21], where link weights were observed to follow a similar trend. This scheme provides a convenient way for the study on network robustness. According to this link weighting scheme, the weight of a node *i* is given by,

$$W_i = \sum_{j \in A(i)} w_{ij} = \sum_{j \in A(i)} (k_i k_j)^{\theta} \qquad (4)$$

In a connected network of size *n* ($n \geq 3$), for an arbitrary link $(i, j) \in E$, it satisfies that $k_i k_j > 1$. Thus, it is easy to find that the link/node weights are all strictly increasing functions of θ . By changing θ , the weights of the links/nodes are changed; then, the routes of information flows change accordingly. As a result, the nodal load B_i carried by a node *i* changes. The maximum nodal load is thus made closely relate to the control parameter θ . In fact, by tuning θ , information flow can be made bias towards large degree nodes ($\theta > 0$) or bias towards low degree nodes ($\theta < 0$). When $\theta = 0$, the network becomes nonweighted (all the links have the same weight), information flow is solely affected by network topology.

In the next section, through computations and simulations, we shall demonstrate that by properly tuning θ , not only the maximum nodal load can be greatly reduced under the same incoming traffic, but also network traffic capacity in terms of packet generation rate can be maximally increased simultaneously.

3. Computations and Simulations

In this section, the random walk-based local flow distribution strategy, along with the link weighting scheme is implemented. A traffic dynamic model is employed for simulation. The model governs the dynamics of traffic generation, storage, and transmission. Here, the traffic generated is packet. In the model, a discrete time clock *t* is used. The simulation starts from $t = 0$. At each time step, first, a packet is generated at each node with packet generation rate λ. The destination of the packet is randomly and uniformly chosen from among all the other nodes. Each node is endowed with an infinite first-in-first-out (FIFO) queue in which packets are stored waiting to be processed. Then, if the queue is not empty, each node picks up one packet from its queue, and transmits it to one of its neighbors (or adjacent nodes). The probability of a neighbor being chosen for packet transmission is calculated according to Eq. (1). A packet will be eliminated immediately from the system as soon as it reaches its destination. To enhance the routing efficiency, we add two modifications in the simulation. First, if node A receives a packet from node B, it memorizes this information, and would not send the packet back to node B at next time step. Second, for routing a packet, each node first performs a local search among its neighbors. If the destination of the packet is found to be among the node's neighbors, it will be delivered directly to its destination.

The SF networks are used as the underlying networks. They are constructed based on the Barabasi-Albert (BA) model proposed in [22]. A SF network has a power law degree distribution such that most nodes have very low degrees, but a few nodes (called hubs) could have extremely high degrees; therefore, the structure of the SF network is heterogeneous. If the SF network is nonweighted, links to and from hub nodes tend to be used more frequently than other links in the network. However, by tuning the weight parameter θ , the amount of traffic passing through either hub nodes or low degree nodes can be adjusted accordingly. Both computational and simulation results are provided next. Each data obtained is averaged over ten different network realizations. <*k*> stands for the average nodal degree.

3.1. Computational results

The maximum nodal load is computed according to Eq. (3). The maximum nodal weight is given by,

$$W_{\max} = \max\{W_i, i = 1,2,...,n\} \qquad (5)$$

The total resistance distance R_{tot} is obtained from the Moore-Penrose inverse L^+ of Laplacian matrix L of the graph [17],

$$R_{tot} = 2nTr(L^+) \qquad (6)$$

where L^+ is given by [23],

$$L^+ = (L + \frac{J}{n})^{-1} - \frac{J}{n} \qquad (7)$$

where J is a $n \times n$ matrix with all elements equal to 1. Therefore, the maximum nodal load is obtained as,

$$B_{\max} = nW_{\max}[Tr(L + \frac{J}{n})^{-1} - 1] \qquad (8)$$

By tuning θ, we find that there exists an optimal weight parameter θ_{opt} at which the maximum nodal load reaches the lowest (shown in Fig. 1). The reason lies in that $B_{\max}$ is the product of $W_{\max}$ and R_{tot} (according to Eq. (3)); $W_{\max}$ is a strictly increasing function of θ (shown in Fig. 2 (a)); while since R_{tot} is a monotone decreasing function of link weights, R_{tot} is also a monotone decreasing function of θ (shown in Fig. 2 (b)). Therefore, there exists an optimal $\theta = \theta_{opt}$, where the maximum nodal load is minimal. With the optimal link weighting, the maximum nodal load decreases about 70% compared with that computed in the corresponding nonweighted networks. By reducing the maximum nodal load to the lowest, the probability of the onset of congestion and cascades can be significantly reduced.

Next, by employing the traffic dynamic model, coupled with the random walk-based local flow distribution strategy and the link weighting scheme, we shall demonstrate through simulation that under the optimal link weighting, a global traffic optimization can be obtained based on local flow distribution. As a result, not only the maximum nodal load is reduced to the lowest, but also network traffic capacity in terms of packet generation rate reaches maximal simultaneously. Better understanding shall be gained on network robustness against congestion and cascades through our simulation.

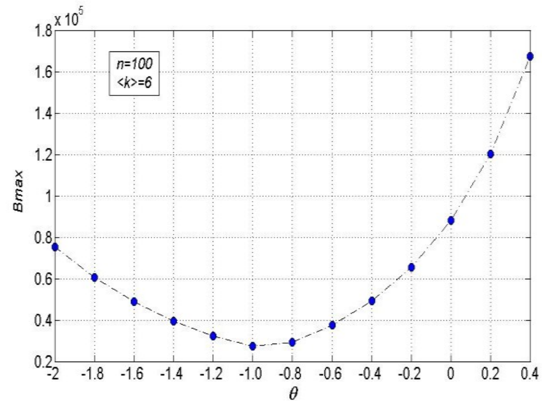

Fig. 1 Computational results of $B_{\max}$ as a function of θ

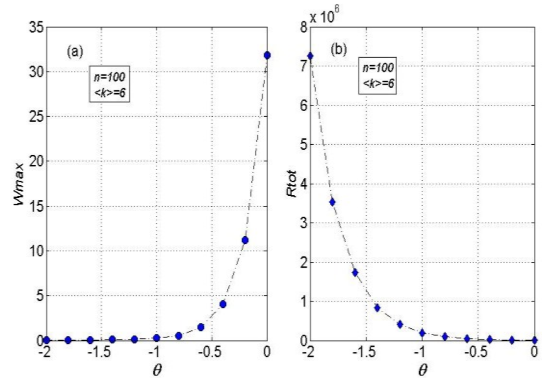

Fig. 2 (a) Maximum nodal weight as a function of θ. (b) Total resistance distance as a function of θ

3.2 Simulation results

Fig. 3 presents the simulation results for $B_{\max}$ as a function of θ, the inset of Fig. 3 shows the corresponding computational results for comparison. We observe from Fig. 3 that the simulation results match the computational results nicely. The optimal weighting parameter θ_{opt} is found to be around -0.9. Fig. 4 displays the average path length $< l >$ of random walk routes as a function of θ. It shows that with the increase in θ, increasing traffic tends to flow through hub nodes, which is demonstrated by the decrease in the average path length.

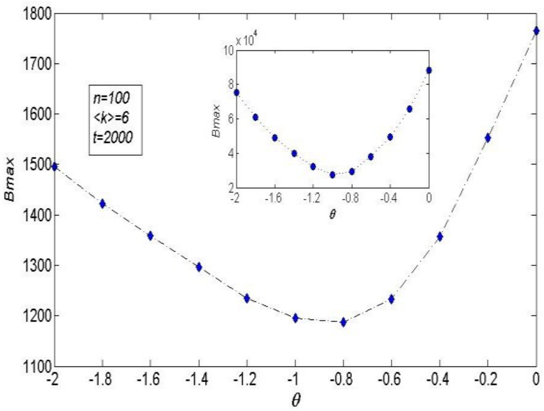

Fig. 3 Simulation results of $B_{\max}$ as a function of θ

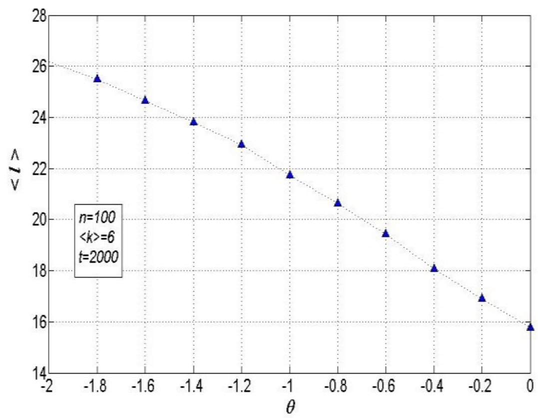

Fig. 4 Average path length as a function of θ

According to the random walk-based local flow distribution strategy and the link weighting scheme implemented, when $\theta < \theta_{opt}$, a packet has a high probability of passing through low degree nodes because of the high weights of the links connecting them. As a result, a low degree node carries more traffic than a hub node. The node that carries maximum load is found among the low degree nodes. However, with the increase in θ, the number of packets passing through hub nodes starts to increase because the weights of the links connecting to hub nodes increase faster than those connecting low degree nodes. With further increase in θ such that $\theta > \theta_{opt}$, more traffic is found to be carried by hub nodes, and the node that carries maximum load is found among the hub nodes. At $\theta = \theta_{opt}$, a global traffic optimization is achieved (the traffic load is balanced), and the maximum nodal load reaches the lowest under the same incoming traffic. The reason lies in that the introduction of the optimal link

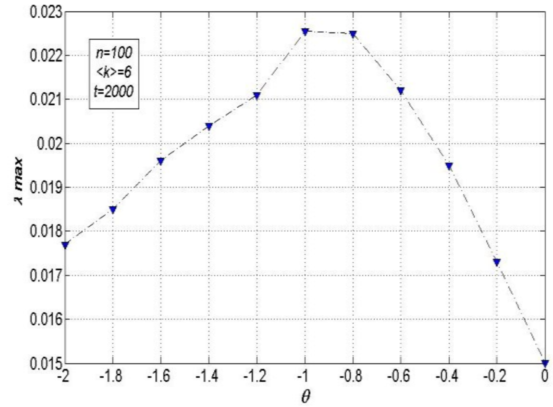

Fig. 5 Maximum packet generation rate as a function of θ

weighting is able to counterbalance the heterogeneity in network structure so as to make traffic flow distribution as uniform as possible. The maximum nodal load decreases about 70% compared with that realized in the corresponding nonweighted networks. The result matches very well the corresponding computational result.

As long as link weights are assigned, it has been found that there exists a maximal packet generation rate $\lambda_{\max}$ beyond which network state changes from traffic free flow to congestion. Therefore, $\lambda_{\max}$ characterizes the traffic capacity of the network. By tuning θ, we observe that $\lambda_{\max}$ changes accordingly. At $\theta = \theta_{opt}$, $\lambda_{\max}$ reaches the highest (shown in Fig. 5), indicating the maximal amount of traffic that the network is capable of handling without causing congestion. With the optimal link weighting, network traffic capacity increases about 60% compared with that realized in the corresponding nonweighted networks. This improvement in network traffic capacity greatly enhances network robustness against congestion and cascading failures.

4. Conclusions

We have investigated network robustness against congestion and cascading failures in weighted complex networks from a design perspective. A random walk-based local flow distribution strategy and a link weighting scheme have been implemented. The link weighting scheme that relates the weights of individual links to local network structure has been inspired from observations in various manmade and natural systems [20-21]. However, whether these systems are tuned to be robust against congestion and cascades is an open question. Through our investigations on the defense of congestion and cascading failures, we find that in weighted scale-free networks, for uniform traffic flow between all source-destination pairs and identical node capacities, properly tuning flow bias toward low degree nodes counterbalances the heterogeneity in network structure such that the maximum nodal load reaches the lowest under the same incoming traffic. The largely decreased maximum nodal load helps effectively defend against the onset of congestion and cascading failures. Our study provides an important addition to the literature on the control and defense of congestion and cascades.

References

[1] J.-W. Wang, L.-L. Rong, "Cascade-based attack vulnerability on the US power grid," *Safety Science 47, Elsevier*, 2009.

[2] W.-X Wang and G. Chen, "Universal robustness characteristic of weighted networks against cascading failure," *Physical Review E* 77, 026101 (2008).

[3] Z.-X. Wu, G. Peng, W.-X. Wang, S. Chan, and E. W. Wong, "Cascading failure spreading on weighted heterogeneous networks," *Journal of Statistical Mechanics: Theory and Experiment*, 2008.

[4] D. Q. Wei, X. S. Luo, B. Zhang, "Analysis of cascading failure in complex power networks under the load local preferential redistribtuion rule," *Physica A 391, Elsevier*, 2012.

[5] J. Zhang, X. Xu, L. Hong, S. Wang, Q. Fei, "Attack vulnerability of self-organizing networks, " *Safety Science* 50, *Elsevier,* pp.443-447, 2012.

[6] C. J. Tessone, A. Garas, B. Guerra, F. Schweitzer, "How big is too big? Critical shocks for systemic failure cascades," arXiv: 1209.0959, 2012.

[7] Adilson E. Motter, "Cascade control and defense in complex networks," *Physics Review Letters* 93, 098701, 2004.

[8] H. J. Sun, H. Zhao, J. J. Wu, "A robust matching model of capacity to defense cascading failure on complex networks," *Physica A* 387, *Elsevier*, pp.6431-6435, 2008.

[9] A. Asztalos, S. Sreenivasan, B. K. Szymanski, and G. Korniss, "Distributed flow optimization and cascading effects in weighted complex networks," *European Physical Journal B*, 85(8) (2012).

[10] R. Yang, W. Wang, Y. Lai, and G. Chen, "Optimal weighting scheme for suppressing cascades and traffic congestion in complex networks," *Physical Review E* 79, 026112 (2009).

[11] A. Shaikh, L. Kalampoukas, A. Varma, and R. Dube, "Routing stability in congested networks: Experimentation and analysis," *Proc. ACM SIGCOMM '00*, pp.163–174, Stockholm, Sweden, 2000.

[12] L. A. N. Amaral, A. Scala, M. Barthelemy, H. E. Stanley, "Classes of small world networks," *Proceedings of the National Academy of Sciences (PNAS)*, Vol. 97, No. 21, pp.11149-11152, 2000.

[13] P. Hines, S. Blumsack, "A centrality measure for electrical networks," *Proceedings of 41st Hawai International Conference on System Sciences*, pp. 185-192, Waikoloa, USA, 2008.

[14] Albert-László Barabási, "Scale-free networks: a decade and beyond," *Science*, Vol. 325, pp.412-413, 2009

[15] X. Ling, M.-B Hu, W.-B. Du, R. Jiang, Y.-H. Wu, Q.-S. Wu, "Bandwidth allocation strategy for traffic systems of scale-free networks," *Physics Letter A 374, Elsevier*, pp.4825-4830, 2010.

[16] G. Korniss, "Synchronization in weighted uncorrelated complex networks in a noisy environment: optimization and connections with transport efficiency," *Physical Review E* 75, 051121 (2007).

[17] A. Tizghadam, A. Leon-Garcia, "A graph theoretical approach to traffic engineering and network control problem," *The IEEE 21st International Teletraffic Congress* (ITC), pp. 1-8, 2009.

[18] A. Ghosh, S. Boyd, and A. Saberi, "Minimizing effective resistance of a graph," *Proceedings of the 17th International Symposium on Mathematical Theory of Networks and Systems*, pp.1185-1196, Kyoto, Japan, 2006.

[19] A. Tizghadam, A. Leon-Garcia, "Robust network planning in nonuniform traffic scenarios," *Computer Communications, Elsevier*, pp. 1-14, 2011.

[20] A. Barrat, M. Barthelemy, R. Pastor-Satorras, A. Vespignani, "The architecture of complex weighted networks," *Proceedings of the National Academy of Sciences (PNAS)*, USA, Vol. 101, No. 11, 2004.

[21] P.J. MacDonald, E. Almaas, A.L. Barabasi, "Minimum spanning trees of weighted scale-free networks," *Europhysics Letters*, Vol. 72, No. 2, pp.308, 2005.

[22] Albert-László Barabási and Réca Albert, "Emergence of scaling in random networks," *Science*, Vol. 286, No. 5439, pp. 509-512, 1999.

[23] Dennis S. Bernstein. *Matrix Mathematics*. Prinston University Press, 2005.