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# APPLICATION OF EQUIVALENCE TECHNIQUE IN LINEAR GRAPH THEORY TO REDUCTION PROCESS IN A POWER SYSTEM*

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## Summary

*This paper deals with the application of a new equivalence technique in linear graph theory to the reduction process in a power system, treated as a linear graph with oriented elements. The formulation of equations in the nodal form, selection of suitable subgraphs or zones and computation of data for individual zones without ever combining them into submatrices for the whole system enables the reduction of the power system into its final daisy equivalent. An illustrative example is worked out for a typical power system with eight generating stations and thirty six elements.*

## Notations

$A$ = incidence matrix,

$A_{mn}$ = partitioned submatrix of the $A$-matrix,

$A_{mn}'$ = transposition of the partitioned submatrix-$A_{mn}$,

$Y_D$ = $D$-graph admittance matrix,

$Y_E$ = $E$-graph admittance matrix,

$Y_{EG}$ = $E$-graph admittance matrix for the final generator daisy,

$Y_S$ = admittance matrix of $S$-graph elements,

$I_E$ = current matrix of $E$-graph elements,

$I_S$ = current matrix of $S$-graph,

$V_E$ = node voltages of $E$-graph,

$V_S$ = node voltages of $S$-graph,

$v_D$ = number of vertices in $D$-graph,

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$v_J$ = number of $J$-vertices,

$u$ = unit matrix, and

$U$ = $U$-vertex.

## 1. Introduction

The single-phase representation of a power system can be correlated with an oriented linear graph where all buses become vertices and lines, transformers, loads, generators, etc., become elements of the graph. The theory developed in the paper[1] can then be applied to the power system to obtain its driving point and transfer admittance matrix which can be used for load flow and stability studies of the system. These admittances are obtained by a reduction process applied to the graph of the power system as described below.

## 2. Reduction process

Subgraphs called 'zones' are selected from the graph of the power system such that (i) all elements of a set which are coupled amongst themselves are in the same zone, and (ii) zones are joined at buses and not through elements. The vertices common between zones are designated as $U$-vertices. The ground vertex will also be a $U$-vertex and will be comon to all the zones. The elements incident to the ground bus are oriented towards it. The $N$ and $E$ graphs are chosen as Lagrangian trees with ground as star point. The well known nodal voltage method of formulating the equations is adopted. The incidence matrix $A$, is obtained by omitting the ground vertex and this incidence matrix is a special form of seg-matrix.[2] The columns and rows in $A$ are so arranged that the loads and/or generators are in the same sequential order as the vertices to which they are incident resulting in unit sub-matrices corresponding to these loads and/or generators. This simplifies the formulae needed for computation. The individual zones are then separately treated in terms of the corresponding $J$-vertices, $D$-graph and $E$-graph according to the 'new equivalence technique'.

The reduced graph of each zone is designated as a 'daisy'. The reduced graphs of all the zones will then be treated as a whole in terms of the corresponding $J$-vertices, $D$-graph and $E$-graph and the final reduced graph will be designated as the 'all-zone generator daisy'.

## 3. Zone reduction

In a zone, the elements represented by loads and generators are the variables while the other system elements are essentially fixed. The first step in the reduction of a zone is to eliminate all fixed elements retaining all vertices and obtain an all-vertex $E$-graph. For that the $D$-graph of the zone is selected such that it contains only fixed elements. It is also necessary that at least one of the elements of the $D$-graph is a grounded element, say, a shunt capacitor. Any vertex which does not have either a load or a generator incident to it (and also not a $U$-vertex) can be considered as a vertex connected to a zero admittance load or phantom load. For $v_D = v_J$, the corresponding equation for any zone will be

$$\begin{bmatrix} I_S \\ I_{E,i} \end{bmatrix} = \begin{bmatrix} Y_S & 0 \\ 0 & A_{12}\, Y_{D,i}\, A_{12}' \end{bmatrix} \begin{bmatrix} V_S \\ V_{E,i} \end{bmatrix} \qquad (1)$$

with $A$ replacing $S$. From equation (1), the admittance matrix of the all-vertex $E$-graph is given by

$$Y_{E,i} = A_{12}\, Y_{D,i}\, A_{12}' \tag{2}$$

where

$$A = j - 1 \begin{bmatrix} N & D \\ u & A_{12} \end{bmatrix}$$

For each zone, $A_{12}$ is such that :

(i) The rows correspond to all but the ground vertex; and

(ii) The columns correspond to all but variable elements of that zone.

Now, $Y_{D,i}$ is the admittance matrix for the transmission system. The $SE$-graph resulting from the formation of the all vertex equivalent is illustrated in Fig. 1.

The heavy lines represent the coupled $E$-graph elements and the light lines represent the original $S$-elements within the zone (generators, loads and phantom loads). This special form of graph is designated as 'daisy'.

The next step in the process of reduction is to remove load and phantom load vertices. This elimination may be done in one or more steps by repeated application of the relevant equation for $v_D > v_J$ given below :

$$Y_E = A_{12}\, Y_{D,i}\, A_{12}' + A_{12}\, Y_{D,i}\, A_{22}'\, J_{EV} \tag{3}$$

Simplification is possible by means of partitioning of $Y_{D,i}$ $A_{12}$ and $A_{22}$. Let,

$$Y_{D,i} = \begin{bmatrix} Y_{11,i} & Y_{12,i} & 0 \\ Y_{21,i} & Y_{22,i} & 0 \\ 0 & 0 & Y_{L,i} \end{bmatrix} \tag{4}$$

where $Y_{L,i}$ is a diagonal admittance matrix with entries corresponding to the loads incident to the vertices to be eliminated.

$Y_{22,i}$ is having rows corresponding to elements of the preceding $E$-graph which are incident to the vertices to be eliminated.

$Y_{11,i}$ has rows corresponding to all $D$-graph elements.

If all load and phantom load vertices are to be eliminated, $Y_{L,i}$ contains the load admittances with zeros for phantom loads and $Y_{22,i}$ contains the admittances for elements parallel to the loads and phantom loads, calculated in the preceding step.

The submatrices $A_{12}$ and $A_{22}$, of the incidence matrix $A$, to be used in equation (3) are defined by

$$\begin{matrix} \text{Vertices to be retained} \\ \text{Vertices to be eliminated} \end{matrix} \begin{bmatrix} A_{12} \\ A_{22} \end{bmatrix} = \begin{bmatrix} u & 0 & 0 \\ 0 & -u & u \end{bmatrix} \tag{5}$$

where the columns correspond to rows in equation (4).

Substituting equations (4) and (5) in equation (3),

$$\begin{aligned} Y_{E,i} &= [Y_{11,i}] + [Y_{12,i}]\,[U_{EV,i}] \\ &= [Y_{11,i}] + [Y_{12,i}]\,[-(Y_{22} + Y_{L,i})]^{-1}\,[Y_{21,i}] \\ &= [Y_{11,i}] - [Y_{12,i}]\,[Y_{22,i} + Y_{L,i}]^{-1}\,[Y_{21,i}] \end{aligned} \qquad (6)$$

This formula does not involve the incidence matrix.

Programming the computation of the equation involves only partitioning an admittance matrix and operating with the submatrices of the partitioned matrix. The *SE*-graph takes the form shown in Fig. 2 and is further reduced by one more all vertex transformation eliminating the loads incident to *U*-vertices and parallel to generators as described below:

$$Y_{D,i} = \begin{bmatrix} Y_{11,i} & Y_{12,i} & 0 & 0 \\ Y_{21,i} & Y_{22,i} & 0 & 0 \\ 0 & 0 & Y_{33,i} & 0 \\ 0 & 0 & 0 & Y_{44,i} \end{bmatrix} \qquad (7)$$

where the leading $2\times2$ matrix is the admittance matrix for Fig. 2 partitioned into:

(i) Elements incident to generator vertices ($Y_{11,i}$); and

(ii) Elements incident to *U*-vertices ($Y_{22,i}$).

$Y_{33,i}$ is a diagonal matrix of load admittances parallel to the generators and $Y_{44,i}$ is a diagonal matrix of load admittances prallel to the *U*-elements. The incidence matrix for the zone will be

$$A_{12} = \begin{matrix} \\ v_G \\ v_U \end{matrix} \begin{matrix} \begin{matrix} E_G & E_U & L_G & L_U \end{matrix} \\ \begin{bmatrix} u & 0 & u & 0 \\ 0 & u & 0 & u \end{bmatrix} \end{matrix} \qquad (8)$$

where $v_G$ are the generator vertices; $v_U$ are the *U*-vertices; $E_G$, $E_U$ are the *E*-graph elements at generator vertices and *U*-vertices respectively; and $L_G$, $L_U$ are the load elements parallel to the generator and *U*-elements respectively.

Then from equations (2), (7) and (8),

$$Y_{E,i} = \begin{bmatrix} Y_{11,i} + Y_{33,i} & Y_{12,i} \\ Y_{21,i} & Y_{22,i} + Y_{44,i} \end{bmatrix} \qquad (9)$$

A perusal of equation (9) shows that it can be obtained by adding the proper load admittance to the diagonal of $Y_{E,i}$ computed for the *SE*-graph of Fig. 2. The reduced graph will be as shown in Fig. 3. It will contain only the *G* and *U* vertices and will be referred to as the *GU*-graph.

## 4. Reduction of zones

The power system graph will have the form as shown in Fig. 4 when the reduction of all the zones is completed.

The *U*-vertices which form the coupling between zones are eliminated next. A graph of the form shown in Fig. 5, designated as 'all-zone generator daisy equivalent' will then be obtained.

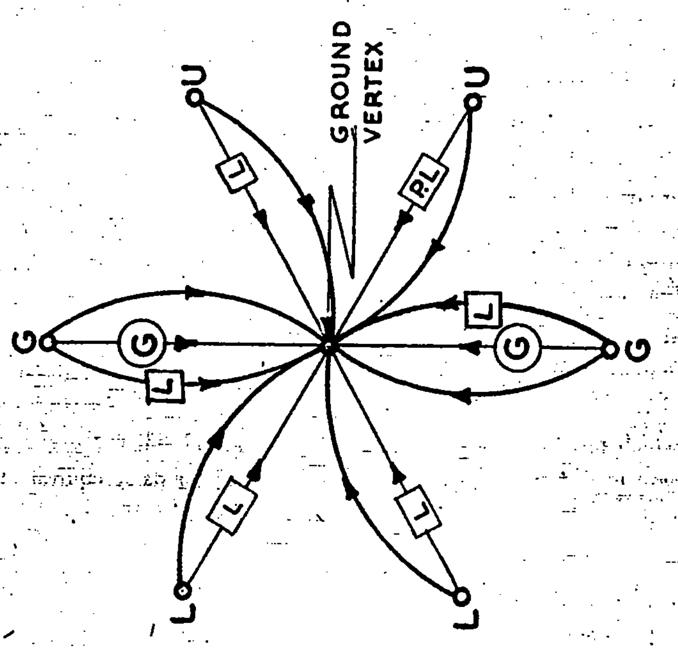


Fig. 1
All-vertex equivalent

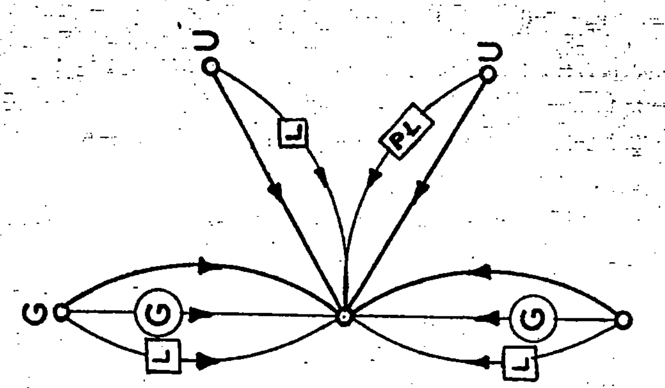


Fig. 2
*G-U* graph with loads

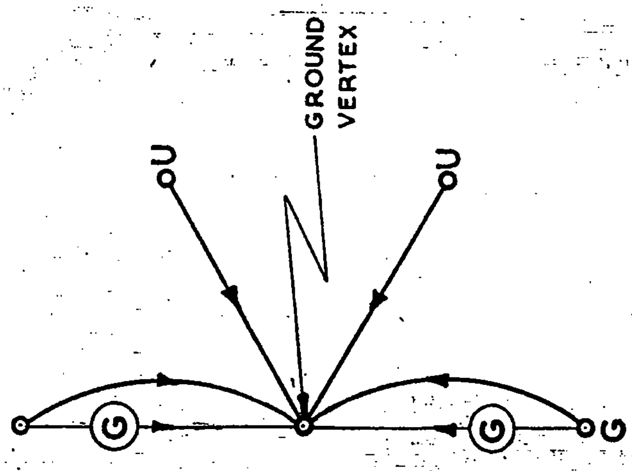


Fig. 3
*G-U* graph

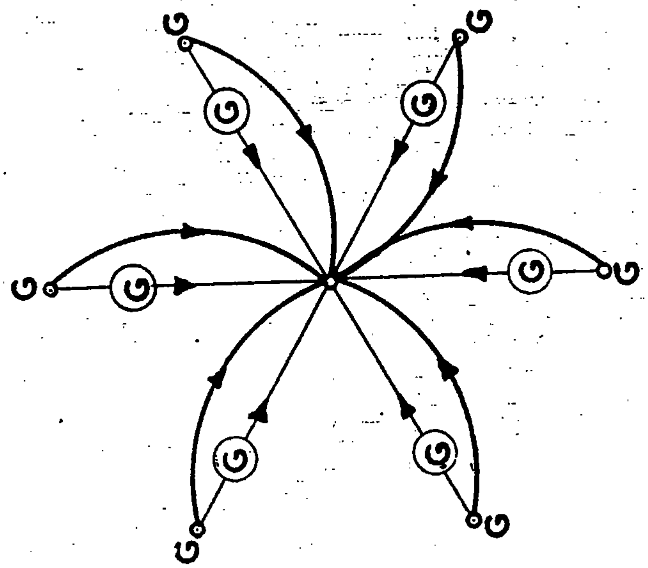


Fig. 5
All-zone generator daisy equivalent

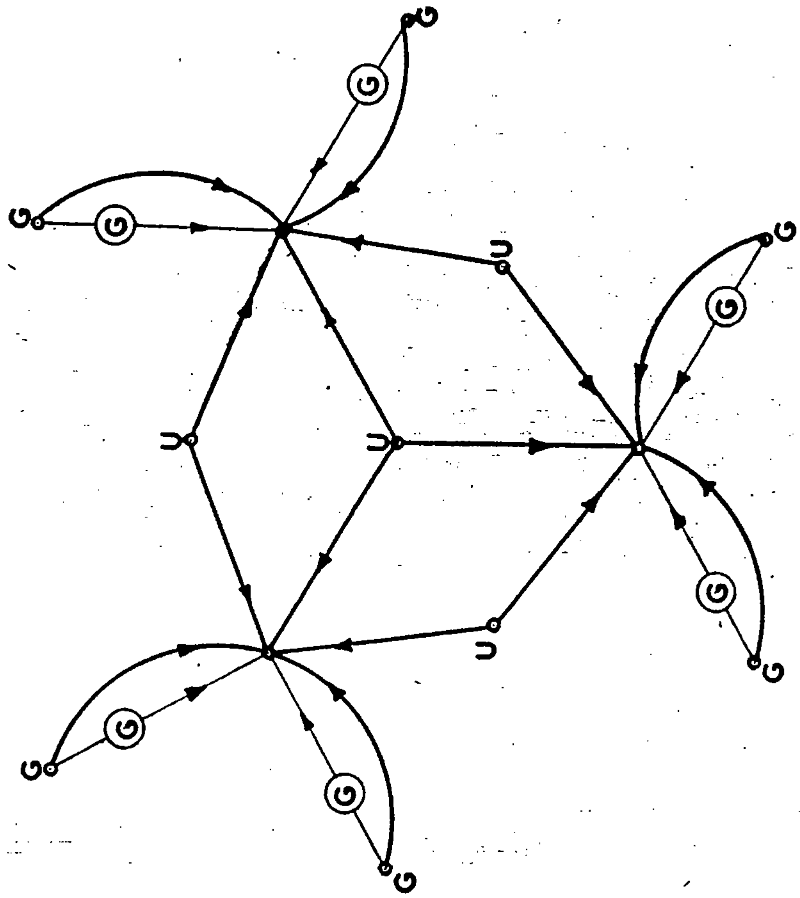


Fig. 4
All-zone *G-U* graph

The general formula for the elimination of *U*-vertices from the all-zone graph of Fig. 4 is given by equation (3). The *D*-graph is all the non-generator elements in the *G-U* graph and the *J*-vertices are the generator vertices of the whole system. The admittance matrix of the all-zone *G-U* graph may be arranged in the partitioned form as :

$$\begin{array}{cc} & \begin{array}{cccccc} \multicolumn{3}{c}{\text{Non-}U\text{-elements}} & \multicolumn{3}{c}{U\text{-elements}} \\ \text{zone 1} & \text{zone 2} & \text{zone N} & \text{zone 1} & \text{zone 2} & \text{zone N} \end{array} \\ Y_D = & \left[\begin{array}{ccc|ccc} Y_{11,1} & & 0 & Y_{12,1} & & 0 \\ & Y_{11,2} & & & Y_{12,2} & \\ & & \ddots & & & \ddots \\ 0 & & Y_{11,n} & 0 & & Y_{12,n} \\ \hline Y_{21,1} & & & Y_{22,1} & & 0 \\ & Y_{21,2} & & & Y_{22,n} & \\ & & \ddots & & & \ddots \\ 0 & & Y_{21,n} & 0 & & Y_{22,n} \end{array}\right] \end{array}$$

$$= \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \tag{10}$$

The incidence matrix will be

$$\begin{array}{ccc} & & \begin{array}{cccccc} \multicolumn{3}{c}{\text{Non-}U\text{-elements}} & \multicolumn{3}{c}{U\text{-elements}} \\ \text{zone 1} & \text{zone 2} & \text{zone N} & \text{zone 1} & \text{zone 2} & \text{zone N} \end{array} \\ \begin{bmatrix} A_{12} \\ A_{22} \end{bmatrix} = & \begin{array}{c} \text{Generator vertices} \\ \\ \\ U\text{-vertices} \end{array} & \left[\begin{array}{ccc|c} u & & 0 & \\ & u & & 0 \\ & & \ddots & \\ 0 & & u & \\ \hline & 0 & & A_{23} \end{array}\right] \end{array} \tag{11}$$

If the set of columns under *U*-elements is ordered properly with respect to zones, then

$$A_{23} = [A_{23,1}\; A_{23,2} \ldots\ldots A_{23,i} \ldots\ldots A_{23,n}] \tag{12}$$

From equation (11),

$$A_{12} = [u \quad 0] \tag{13}$$

$$A_{22} = [0 \;\; A_{23}] \tag{14}$$

The different terms of equation (3) can be expressed after substituting the appropriate submatrices from equations (10) and (11) as follows :

$$A_{12}\, Y_D\, A_{12}' = Y_{11} \tag{15}$$

$$J_{EV} = -[A_{23}\, Y_D\, A_{23}']^{-1}\ [A_{23}\, Y_D\, A_{12}']$$

$$= -[A_{23}\, Y_{22}\, A_{23}']^{-1}\ [A_{23}\, Y_{21}] \tag{16}$$

$$A_{12}\, Y_D\, A_{23}' = [Y_{12}\, A_{23}'] \tag{17}$$

Using equations (15), (16) and (17) in equation (3), we get

$$Y_{EG} = A_{12}\, Y_D\, A_{12}' + [A_{12}\, Y_D\, A_{23}']\, J_{EV}$$

$$= Y_{11} - Y_{12}\, A_{23}'\, [A_{23}\, Y_{22}\, A_{23}']^{-1}\, A_{23}\, Y_{21} \tag{18}$$

Substituting for $Y_{12}$ and $A_{23}'$ from equations (10) and (12) and simplifying,

$$Y_{12}\, A_{23}' = \begin{bmatrix} Y_{12.1}\ A_{23.1}' \\ Y_{12.2}\ A_{23.2}' \\ \cdots\cdots \\ \cdots\cdots \\ Y_{12.i}\ A_{23.i}' \\ \cdots\cdots \\ \cdots\cdots \\ Y_{12.n}\ A_{23.n}' \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \\ \cdot\cdot \\ \cdot\cdot \\ M_i \\ \cdot\cdot \\ \cdot\cdot \\ M_n \end{bmatrix} \tag{19}$$

where

$$M_i = Y_{12.i}\, A_{23.i}' \tag{20}$$

Substituting for $A_{23}$, $Y_{22}$ and $A_{23}'$ from equations (10) and (12) and simplifying,

$$A_{23}\, Y_{22}\, A_{23}' = [A_{23.1}\ \ldots\ A_{23.n}] \begin{bmatrix} Y_{22.1} & & & \\ & Y_{22.2} & & \\ & & \ddots & \\ & & & Y_{22.n} \end{bmatrix} \begin{bmatrix} A_{23.1}' \\ A_{23.2}' \\ \cdot\cdot \\ \cdot\cdot \\ A_{23.n}' \end{bmatrix}$$

$$= B$$

where

$$B = \sum_{i=1}^{i=n} B_i \tag{21}$$

and

$$B_i = A_{23.i}\, Y_{22.i}\, A_{23.i}' \tag{22}$$

Similarly,

$$A_{23}\, Y_{21} = [Y_{12}\, A_{23}']^{-1}$$

$$= [M_1', M_2' \ \ldots\ldots\ M_i' \ \ldots\ldots\ M_n'] \tag{23}$$

Substituting for $Y_{11}$ from equation (10) and equations (19), (21) and (23) in equation (18),

$$Y_{EG} = \begin{bmatrix} Y_{11.1} & & & \\ & Y_{11.2} & & \\ & & \ddots & \\ & & & Y_{11.n} \end{bmatrix} - \begin{bmatrix} M_1 \\ M_2 \\ \cdot \\ \cdot \\ M_n \end{bmatrix} [B]^{-1}\, [M_1'\, M_2' \ \ldots\ldots\ M_n']$$

$$Y_{EG} = \begin{bmatrix} Y_{11.1} - M_1 B^{-1} M_1' & - M_1 B^{-1} M_2' & \ldots & - M_1 B^{-1} M_n' \\ - M_2 B^{-1} M_1' & Y_{11.2} - M_2 B^{-1} M_2' & \ldots & - M_2 B^{-1} M_n' \\ \cdots\cdots & \cdots\cdots & & \cdots\cdots \\ - M_n B^{-1} M_1' & - M_n B^{-1} M_2' & \ldots & Y_{11.n} - M_n B^{-1} M_n' \end{bmatrix} \quad (24)$$

An analysis of the last equation shows that the computation of $Y_{EG}$ can be effected with data computed for individual zones without ever combining them into submatrices for the whole system though equation (24) actually calculates $Y_{EG}$. Thus, the amount of computer storage required can be reduced appreciably.

## 5. Example

An example is worked out to illustrate the application of the reduction technique discussed so far to a power system containing eight generating stations and thirty six elements. Fig. 6 shows the single line diagram of the power system under consideration in which lines are designated by numbers and buses by lower-case letters. The reactances of all elements in per unit on a base of 100 MVA are given in Table 1 (*vide* Appendix).

Resistances of lines are neglected. No loads are considered. Long lines are represented by '*pi*' equivalent with capacitors as shunt branches.

Figs. 7(i), (ii) and (iii) show the three zones into which the power system of Fig. 6 is sub-divided. The results of reduction at the zonal level are given for zone 1 only.

The vertices common to zones 1 and 2 are *g* and *i* and those common to zones 2 and 3 are *i*, *l* and *m*.

The incidence submatrix $A_{12}$, of the transmission system of each zone are given below.

For zone 1,

$$A_{12} \text{ matrix} = \begin{array}{c} \\ a \\ b \\ c \\ d \\ g \\ i \\ e \\ f \\ h \end{array} \begin{array}{c} \begin{array}{cccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \quad [25(a)]$$

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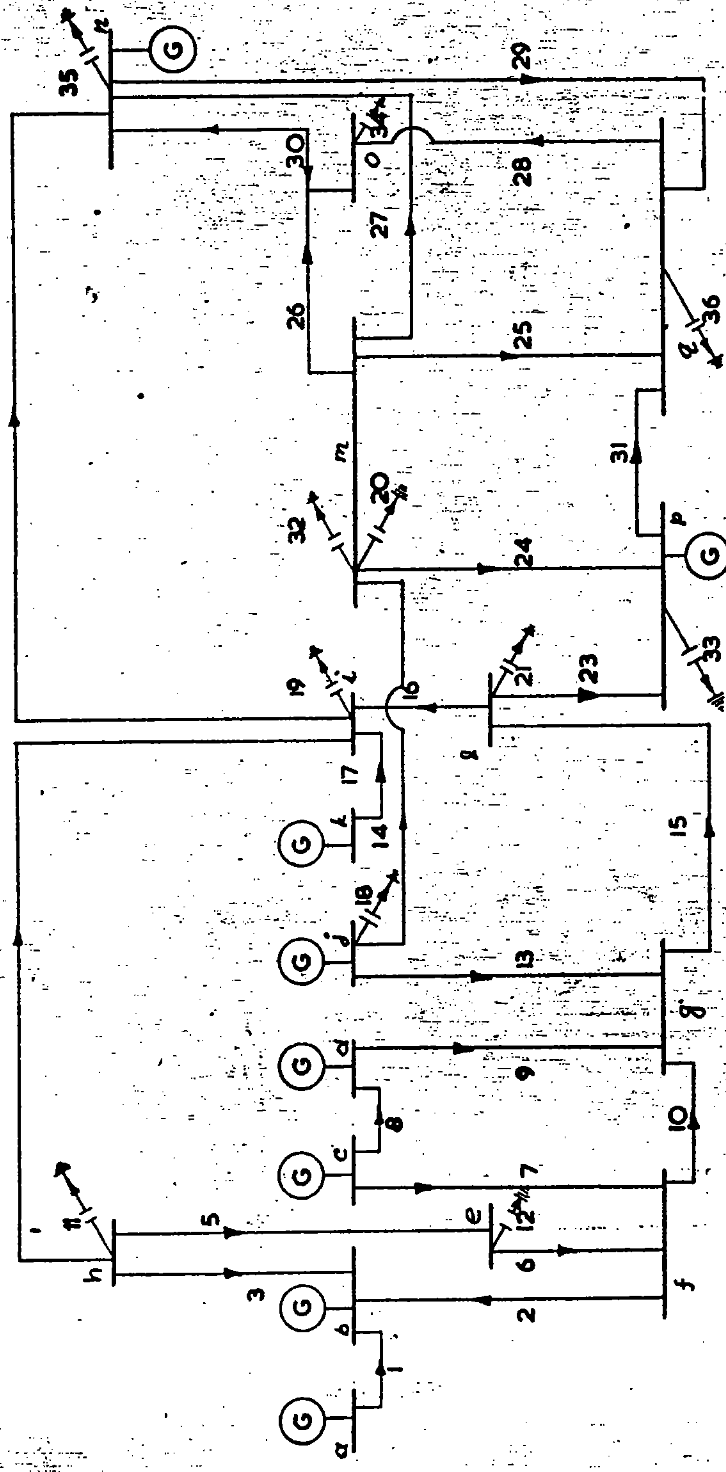
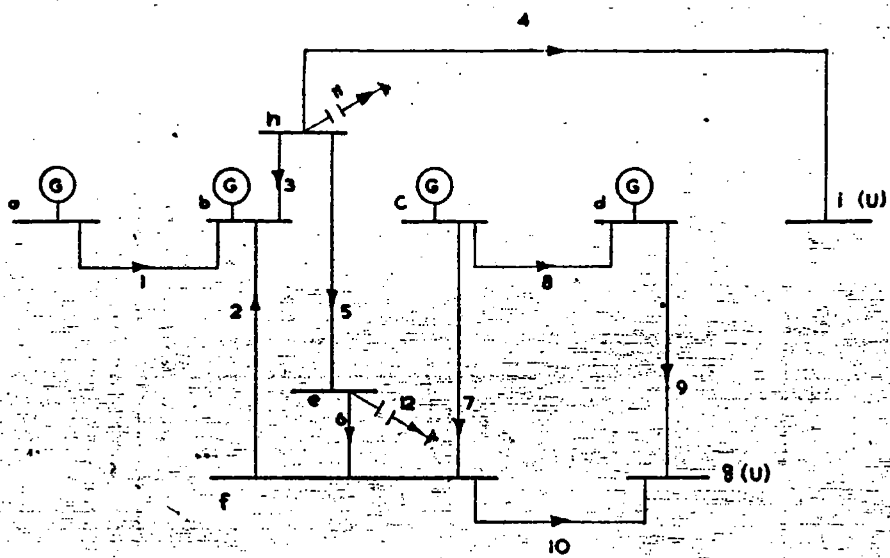
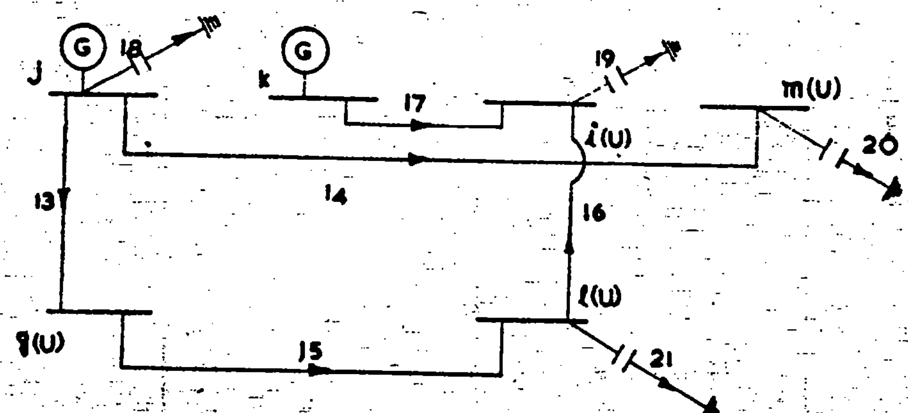



Fig. 6
Power system

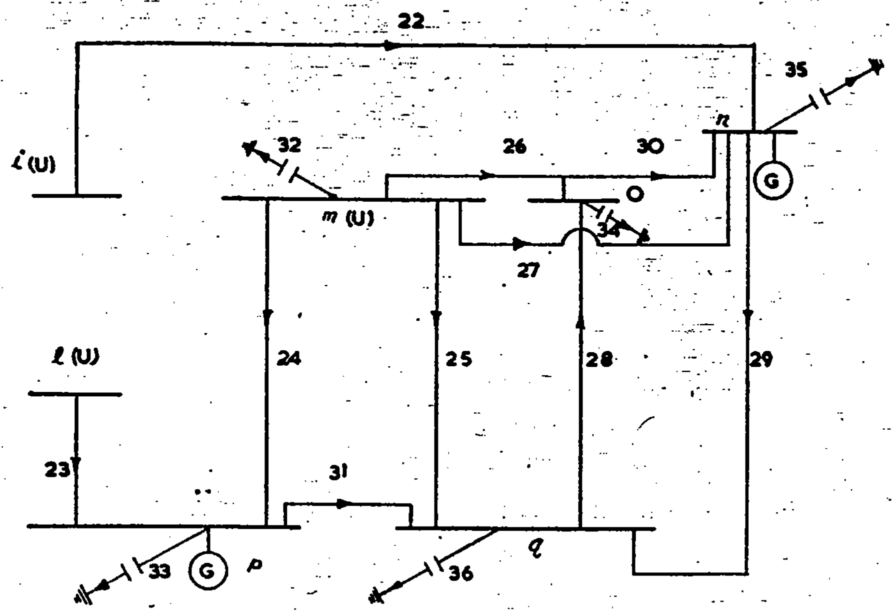



(i)
Zone 1




(ii)
Zone 2

Fig. 7 (*Contd.*)




(iii)
**Zone 3**

**Fig. 7**

**Diagram showing the sub-divided zones of Fig. 6**

For zone 2,

$A_{12}$ matrix =

| | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|
| *j* | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| *k* | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| *g* | −1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| *i* | 0 | 0 | 0 | −1 | −1 | 0 | 1 | 0 | 0 |
| *l* | 0 | 0 | −1 | 1 | 0 | 0 | 0 | 0 | 1 |
| *m* | 0 | −1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |

[25(b)]

For zone 3,

$$
A_{13} \text{ matrix} = \begin{array}{c} \\ p \\ n \\ i \\ l \\ m \\ o \\ q \end{array}
\begin{array}{c}
\begin{array}{ccccccccccccccc} 22 & 23 & 24 & 25 & 26 & 27 & 28 & 29 & 30 & 31 & 32 & 33 & 34 & 35 & 36 \end{array} \\
\left[\begin{array}{ccccccccccccccc}
0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & -1 & -1 & 0 & 0 & -1 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 1
\end{array}\right]
\end{array}
\qquad [25(c)]
$$

Substituting equations 25(a), (b) and (c) and the corresponding $Y_D$ from Table 1, in equation (2), the $E$ admittance matrix of the all-vertex equivalent of each zone is obtained. The all-vertex equivalent graphs of the zones are shown in Figs. 8(i), (ii) and (iii).

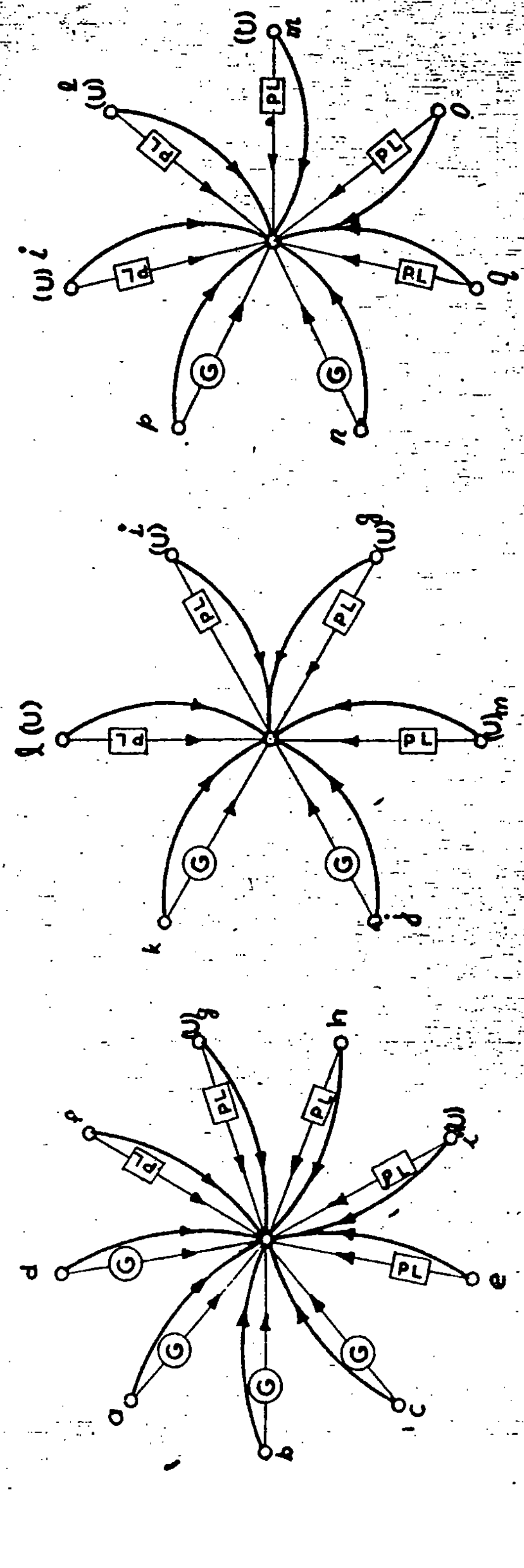
The $E$-admittance matrix of the 'daisy' or all-vertex equivalent of zone 1 is given in Table 2 (in a 'daisy', an element will be referred to by the vertex other than the common vertex to which it is incident and only non-zero elements of the matrix are given in the table).

The next step is the elimination of vertices which are neither $U$-vertices nor generator vertices. For zone 1, such vertices are $e$, $f$ and $h$, and for zone 3 they are $q$ and $o$. For zone 2, there are no such vertices.

The submatrix $Y_{L,i}$ of equation (4) is a zero matrix.

The submatrices $Y_{11,i}$, $Y_{12,i}$, $Y_{22,i}$ for each zone can be obtained from the corresponding all vertex $E$-admittance matrix. Using these submatrices, the $E$-admittance matrix of the $G$-$U$ graph of zones 1 and 3 are calculated. The $G$-$U$ graph for zone 2 is the same as its all-vertex graph. In Figs. 9(i), (ii) and (iii) the $G$-$U$ graphs of the zones are shown. For zone 1, the $E$-admittance matrix of $G$-$U$ graph is given in Table 3 (only non-zero elements of the matrix are given). The all-zone $G$-$U$ graph is shown in Fig. 10.

The next step is the elimination of $U$-vertices from the all-zone $G$-$U$ graph. The incidence matrix $A_{33}$, of all the $U$-elements with partitioning as shown in equation (12) is given by equation (26).




(i) All-vertex equivalent zone 1

(ii) All-vertex equivalent zone 2

(iii) All-vertex equivalent zone 3

Fig. 8

All-vertex equivalent graphs of the zones

$$A_{23} = \begin{array}{c} \\ \\ \\ g \\ i \\ l \\ m \end{array} \begin{array}{cc|cccc|ccc} \multicolumn{2}{c|}{\text{Zone 1}} & \multicolumn{4}{c|}{\text{Zone 2}} & \multicolumn{3}{c}{\text{Zone 3}} \\ \multicolumn{2}{c|}{A_{23,1}} & \multicolumn{4}{c|}{A_{23,2}} & \multicolumn{3}{c}{A_{23,3}} \\ g & i & g & i & l & m & i & l & m \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{array} \tag{26}$$

The submatrices $Y_{11,i}$, $Y_{12,i}$, $Y_{22,i}$ of equation (10) can be obtained for each zone from the $E$-admittance matrix of the corresponding $G$-$U$ graph. Using these submatrices and the submatrices of equation (26), the $M$-matrix of each zone and the $B^{-1}$ matrix can be calculated. Finally, the various submatrices such as $Y_{11,i} - M_1 B^{-1} M_1'$ and $- M_1 B^{-1} M_2'$ of the generator daisy admittance matrix are calculated. The system generator daisy is shown in Fig. 11.

The system generator daisy admittance matrix of the power system shown in Fig. 6 is then given by Table 4 (only non-zero elements of the matrix are given).

## 6. Conclusions

In a symmetrical power system it is always possible to choose subgraphs or zones such that all coupled elements remain within a zone. Then independent reduction of individual zones is feasible. Again in a zone, $S$ and $D$ graphs can be so chosen that there is no coupling between them. This reduces considerably the computer storage and computation work and therefore computer time for obtaining the $E$-admittance matrix of each zone.

The presence of earth in a power system enables the choice of a Lagrangian tree for the $E$-graph resulting in unit submatrices with $+1$ entries in the incidence matrices of the zones. In the reduction of the all-vertex equivalent of Fig. 1 to Fig. 2, the presence of the unit matrices in the incidence matrix enables one to work with submatrices of the $E$-admittance matrix of the all-vertex equivalent itself without the necessity of forming new incidence matrices, thus simplifying the mathematical computations.

In the reduction stage from Fig. 2 to Fig. 3, loads which are parallel elements in Fig. 2 are being eliminated and it was found that the $E$-admittance matrix of Fig. 3 can be obtained by simple addition of the relevant submatrices of the $E$-graph of Fig. 2 without the necessity of going through the reduction procedure.

An analysis of the equation (24) for the $E$-admittance matrix of the all generator daisy of Fig. 5 shows that the computation of the individual entries in the matrix can be effected with the data computed for the individual zones. It is, therefore, not necessary to combine the data from the individual zones to give submatrices and then go through the reduction process to obtain the final $E$-admittance matrix from the all $G$-$U$ graph. Hence the computer storage required for the reduction of a power system will be only that necessary for the reduction of a zone. A power system, however large it may be, can always be zoned such that the reduction of these zones is within the capacity of the available computer.

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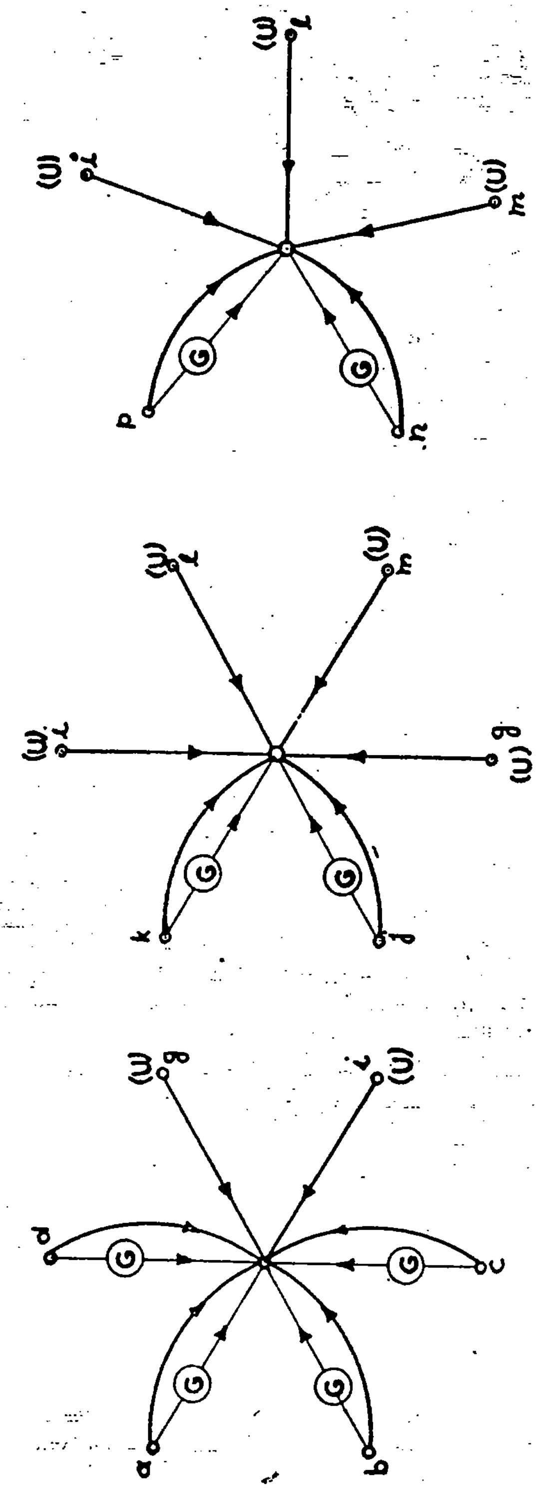


(i) *G-U* graph zone 1

(ii) *G-U* graph zone 2

(iii) *G-U* graph zone 3

**Fig. 9**

***G-U* graphs of the zones**

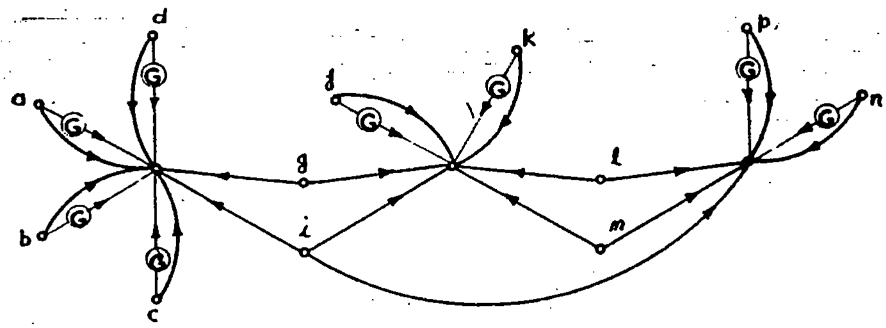


Fig. 10
All-zone *G-U* graph

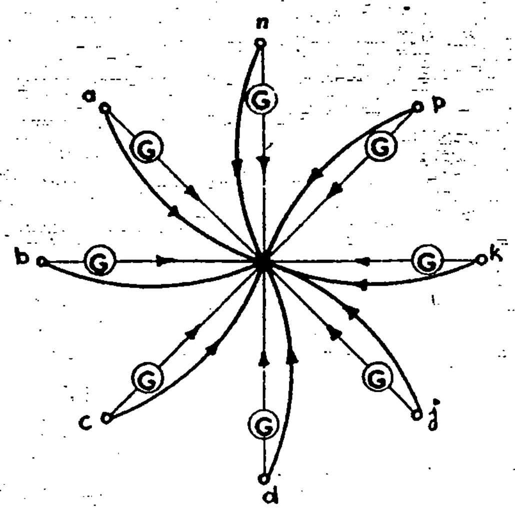


Fig. 11
System generator daisy

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## Appendix

Table 1

Per unit reactance of elements

| Element number | Reactance in per unit | Element number | Reactance in per unit |
|---|---|---|---|
| 1 | 0.010 | 19 | —7.68 |
| 2 | 0.078 | 20 | —25.00 |
| 3 | 0.070 | 21 | —20.00 |
| 4 | 0.132 | 22 | 0.35 |
| 5 | 0.029 | 23 | 0.244 |
| 6 | 0.092 | 24 | 0.442 |
| 7 | 0.080 | 25 | 0.386 |
| 8 | 0.022 | 26 | 0.456 |
| 9 | 0.198 | 27 | 0.680 |
| 10 | 0.356 | 28 | 0.511 |
| 11 | —7.680 | 29 | 0.260 |
| 12 | —6.660 | 30 | 0.208 |
| 13 | 0.094 | 31 | 0.056 |
| 14 | 0.176 | 32 | —20.000 |
| 15 | 0.220 | 33 | —20.000 |
| 16 | 0.092 | 34 | —20.000 |
| 17 | 0.024 | 35 | — 7.500 |
| 18 | —25.000 | 36 | —20.000 |

Table 2

*E*-admittance matrix of all-vertex equivalent for zone I

| Row | Column | Reactance in per unit |
|---|---|---|
| 1 | 1 | −0.10000000 E + 03* |
| 1 | 2 | 0.10000000 E + 03 |
| 2 | 1 | 0.10000000 E + 03 |
| 2 | 2 | −0.12710622 E + 03 |
| 2 | 8 | 0.12820513 E + 02 |
| 2 | 9 | 0.14285714 E + 02 |
| 3 | 3 | −0.57954545 E + 02 |
| 3 | 4 | 0.45454545 E + 02 |
| 3 | 8 | 0.12500000 E + 02 |
| 4 | 3 | 0.45454545 E + 02 |
| 4 | 4 | −0.50505050 E + 02 |
| 4 | 5 | 0.50505051 E + 01 |
| 5 | 4 | 0.50505051 E + 01 |
| 5 | 5 | −0.78594939 E + 01 |
| 5 | 8 | 0.28089888 E + 01 |
| 6 | 6 | −0.75757576 E + 01 |
| 6 | 9 | 0.75757576 E + 01 |
| 7 | 7 | −0.45202174 E + 02 |
| 7 | 8 | 0.10869565 E + 02 |
| 7 | 9 | 0.34482759 E + 02 |
| 8 | 2 | 0.12820513 E + 02 |
| 8 | 3 | 0.12500000 E + 02 |
| 8 | 5 | 0.28089888 E + 01 |
| 8 | 7 | 0.10869565 E + 02 |
| 8 | 8 | −0.38999066 E + 02 |
| 9 | 2 | 0.14285714 E + 02 |
| 9 | 6 | 0.75757576 E + 01 |
| 9 | 7 | 0.34482759 E + 02 |
| 9 | 9 | −0.56214022 E + 02 |

* $-0.10000000\,E+03 = -0.10000000 \times 10^3 = -100.00000$

Table 3

*E*-admittance matrix of *G-U* graph for zone 1

| Row | Column | Reactance in per unit |
|---|---|---|
| 1 | 1 | −0.10000000 E + 03 |
| 1 | 2 | 0.10000000 E + 03 |
| 2 | 1 | 0.10000000 E + 03 |
| 2 | 2 | −0.11202121 E + 03 |
| 2 | 3 | 0.61538772 E + 01 |
| 2 | 5 | 0.13828937 E + 01 |
| 2 | 6 | 0.46525300 E + 01 |
| 3 | 2 | 0.61538771 E + 01 |
| 3 | 3 | −0.53370613 E + 02 |
| 3 | 4 | 0.45454545 E + 02 |
| 3 | 5 | 0.10300973 E + 01 |
| 3 | 6 | 0.77021611 |
| 4 | 3 | 0.45454545 E + 02 |
| 4 | 4 | −0.50505050 E + 02 |
| 4 | 5 | 0.50505051 E + 01 |
| 5 | 2 | 0.13828938 E + 01 |
| 5 | 3 | 0.10300973 E + 01 |
| 5 | 4 | 0.50505051 E + 01 |
| 5 | 5 | −0.76280114 E + 01 |
| 5 | 6 | 0.17308227 |
| 6 | 2 | 0.46525300 E + 01 |
| 6 | 3 | 0.77021614 |
| 6 | 5 | 0.17308227 |
| 6 | 6 | −0.55274277 E + 01 |

Table 4

System generator daisy admittance matrix

| Row | Column | Reactance in per unit |
|---|---|---|
| 1 | 1 | —0.10000000 E + 03 |
| 1 | 2 | 0.10000000 E + 03 |
| 2 | 1 | 0.10000000 E + 03 |
| 2 | 2 | —0.11150521 E + 03 |
| 2 | 3 | 0.62991006 E + 01 |
| 2 | 4 | 0.37697712 |
| 2 | 5 | 0.79405820 |
| 2 | 6 | 0.36967688 E + 01 |
| 2 | 7 | 0.27450750 |
| 2 | 8 | 0.25349272 |
| 3 | 2 | 0.62991005 E + 01 |
| 3 | 3 | —0.53306978 E + 02 |
| 3 | 4 | 0.45704142 E + 02 |
| 3 | 5 | 0.52574853 |
| 3 | 6 | 0.68851726 |
| 3 | 7 | 0.85122276 E + 01 |
| 3 | 8 | 0.47212612 E + 01 |
| 4 | 2 | 0.37697713 |
| 4 | 3 | 0.45704142 E + 02 |
| 4 | 4 | —0.49325009 E + 02 |
| 4 | 5 | 0.24856182 E + 01 |
| 4 | 6 | 0.48241346 |
| 4 | 7 | 0.25013008 |
| 4 | 8 | 0.33079780 E + 01 |
| 5 | 2 | 0.79405820 |
| 5 | 3 | 0.52574853 |
| 5 | 4 | 0.24856182 E + 01 |
| 5 | 5 | —0.85991940 E + 01 |
| 5 | 6 | 0.10161475 E + 01 |
| 5 | 7 | 0.24167767 E + 01 |
| 5 | 8 | 0.14634906 E + 01 |
| 6 | 2 | 0.36967688 E + 01 |
| 6 | 3 | 0.68861726 |
| 6 | 4 | 0.48241346 |

Table 4 (*Contd.*)

| Row | Column | Reactance in per unit |
|---|---|---|
| 6 | 5 | 0.10161475 E + 01 |
| 6 | 6 | —0.97424790 E + 01 |
| 6 | 7 | 0.18450414 E + 01 |
| 6 | 8 | 0.21890872 E + 01 |
| 7 | 2 | 0.27450750 |
| 7 | 3 | 0.85122276 E + 01 |
| 7 | 4 | 0.25013008 |
| 7 | 5 | 0.24167767 E + 01 |
| 7 | 6 | 0.18450414 E + 01 |
| 7 | 7 | —0.93220210 E + 01 |
| 7 | 8 | 0.46007102 E + 01 |
| 8 | 2 | 0.25349272 |
| 8 | 3 | 0.47212612 E + 01 |
| 8 | 4 | 0.33079780 E + 01 |
| 8 | 5 | 0.14634906 E + 01 |
| 8 | 6 | 0.21890872 E + 01 |
| 8 | 7 | 0.46007103 E + 01 |
| 8 | 8 | —0.83762390 E + 01 |