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SYNTHESIS OF MULTIVARIABLE NETWORKS

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Abstract

In this paper we consider the synthesis of certain classes of multivariable networks.

1. INTRODUCTION

In this paper we consider the following aspects of the problem of synthesis of multivariable networks:

(1) A graph-theoretic approach for the synthesis of driving-point functions of multivariable networks.
(2) Synthesis of a class of multivariable n-port networks.

2. A GRAPH-THEORETIC APPROACH FOR THE SYNTHESIS OF MULTIVARIABLE NETWORKS

In this section we give a graph-theoretic approach for the synthesis of driving-point functions of a class of multivariable networks.

Consider a network N. Let G be the linear graph of N. Let there be n edges in N. Let the admittance of edge e_j be equal to $K_j p_j$. We assume that p_j's are distinct. Let there be an edge in N connecting vertices 1 and 2, where the vertices 1 and 2 form the input-terminal-pair. Without any loss of generality we may denote thie edge by e_1. We further assume that there are no parallel edges in N.

The driving-point admittance Y of N across the terminal-pair (1,2) is given by [1].

$$Y = \frac{V(Y)}{W_{1,2}(Y)} \tag{1}$$

where

$V(Y) = \sum$ tree-admittance products (all trees)

and

$W_{1,2}(Y) = \sum$ admittance products of 2-trees $T_{2_{1,2}}$.

We note that Y is a positive real function of the variables p_i's.

Let N_s denote the network obtained after short-circuiting the edge e_1 in N. Then a 2-tree $T_{2_{1,2}}$ of N is also a tree of N_s.

Since no tree of N_s will contain the edge e_1 the variable p_1 will not be present $W_{1,2}(Y)$.

Each term in V(Y) corresponds to a tree in G and can be written as

$$K_{i_1 i_2 \ldots i_{v-1}} \sum_{k=1}^{v-1} p_{i_j}$$

where v is the number of vertices in N and

$$K_{i_1 i_2 \ldots i_{v-1}} = \sum_{j=1}^{v-1} K_{i_j}$$

The tree corresponding to this term will consist of the edges $e_{i_1}, e_{i_2}, \ldots e_{i_{v-1}}$.

Each term in $W_{1,2}(Y)$ corresponds to a 2-tree of N and can be written as

$$K_{i_1 i_2 \ldots i_{v-2}} \sum_{j=1}^{v-2} p_{i_j}$$

The 2-tree corresponding to this term will consist of the edges $e_{i_1}, e_{i_2}, \ldots e_{i_{v-2}}$.

We wish to obtain the graph G of the network N realizing Y and also the constants K_i's associated with the edges of G. Towards this end we proceed as follows.

Consider any term in V(Y). Let the tree T corresponding to this term consist of the edges $e_{i_1}, e_{i_2}, \ldots e_{i_{v-1}}$. Let C_f be the fundamental cutset matrix of G with respect to T. Let the entry of C_f at the intersection of the row and column corresponding to the edges e_i and e_j be denoted by C_{ij}.

Consider any chord e_x. We wish to determing $C_{i_j x}$.

If there exists a term in V(Y) such that the tree T_x corresponding to this is equal to

$$(T - e_{i_j}) \cup e_x$$

then $C_{i_j x} = 1$, otherwise $C_{i_j x} = 0$. This result is a consequence of the fact that $C_{i_j x} = 1$ iff the fundamental circuit corresponding to the chord e_x contains the branch e_{i_j} of T. Thus, in this way all the columns of C_f corresponding to the chords of T can be determined. It is well known that the submatrix of C_f formed by the columns corresponding to the chords of T is a unit matrix. Thus following this procedure, the fundamental cutset matrix C_f of the graph G of the network N realizing Y can be determined. Using C_f, the graph G can be constructed. The terminals of edge e_1 can be identified as the input-terminals of the required network N.

It now remains to determine the constants K_i's associated with the edges of G. To determine these constants we proceed as follows.

We note that in any non-degenerate network for every edge e_i, there exists a path between the input terminals containing e_i. In view of this, we can conclude that for every edge e_{i_j} there exists a tree $T = e_{i_1} \cup e_{i_2} \ldots e_{i_j} \ldots \cup e_{i_{v-1}}$ such that $(T - e_{i_j})$ is a 2-tree. The term K_a^* of V(Y) corresponding to T is given by

$$K_a^* = K_{i_1 i_2 \ldots i_{v-1}} \prod_{j=1}^{v-1} p_{i_j}$$

and the term of $W_{1,2}(Y)$ corresponding to $(T - e_{i_j})$ is given by

$$K_b^* = K_{i_1 i_2 \ldots i_{j-1} i_{j+1} \ldots i_{v-1}} \prod_{\substack{k=1 \\ k\neq j}}^{v-1} p_{i_k}$$

Since

$$K_{i_1 i_2 \ldots i_{v-1}} = \prod_{j=1}^{v-1} K_{i_j}$$

and

$$K_{i_1 i_2 \ldots i_{j-1} i_{j+1} \ldots i_{v-1}} = \prod_{\substack{k=1 \\ k\neq j}}^{v-1} K_{i_k}$$

we get

$$\frac{K_a^*}{K_b^*} = K_{i_j} p_{i_j}$$

Thus the constant K_{i_j} associated with the edge e_{i_j} can be determined as described above. Thus all the constants K_i's can be determined.

This completes our discussion of the procedure to follow in realizing Y.

We summarize our discussions as follows:

i) a) Consider any term in V(Y). Let the tree T corresponding to this term consist of the edges $e_{i_1}, e_{i_2} \ldots e_{i_{v-1}}$. Let $e_x \notin T$. Then

$C_{i_j x} = 1$ if there exists a tree T_x such that

$$T_x = (T - e_{i_j}) \cup e_x.$$

b) The submatrix of C_f formed by the columns corresponding to the branches of T is a unit matrix.

Thus the fundamental cutset matrix C_f of G can be determined.

ii) Consider any term in V(Y) corresponding to a tree T. Let this be equal to K_a^*. Let $e_x \notin T$.

Let $(T - e_x)$ be a 2-tree. Let the term in $W_{1,2}(Y)$ corresponding to this 2-tree be K_b^*. Then $\frac{K_a^*}{K_b^*} = K_x p_x$. Thus all the constants K_i's can be determined.

We note that to determine graph G all the terms in V(Y) will not be required. Hence one has to check, after obtaining G, whether all the trees and 2-trees $T_{2_{1,2}}$ of G are represented in V(Y) and $W_{1,2}(Y)$ respectively.

Extension of the above procedure to synthesize driving-point impedance functions is straightforward.

Further, if the admittances of some of the elements of N are known to be of the form $\frac{K_i}{p_i}$, then synthesis should be carried out using a modified function Y' which is obtained from Y by making the substitution $p_i = \frac{1}{p_i'}$.

3. SYNTHESIS OF A CLASS OF MULTIVARIABLE n-PORT NETWORKS

In this section, we give a simple sufficient condition for the synthesis of a class of multivariable n-port networks.

Consider an n-port network N. Let the entries of Y, the short-circuit admittance matrix of N be functions of the m-variables p_i, $i = 1,\ldots,m$. Let Y be decomposable as follows

$$Y = \sum_{i=1}^{m} \frac{k_i\ p_i}{p_i + \sigma_i} K_i$$

Procedures are available to obtain such a decomposition if one exists [2].

We note that each K_i is a real symmetrix matrix. Let K_i be realizable as the short-circuit conductance matrix of an n-port network N_i^*. Let N_i^* contain no negative conductances.

The network N_i realizing $\frac{k_i\ p_i}{p_i + \sigma_i} K_i$ can be obtained from N_i^* by replacing each conductance g of N_i by a series combination of an admittance $\frac{k_i\ g\ p_i}{\sigma_i}$ and a conductance $k_i g$.

If all the networks N_i^* have the same modified cutset matrix, then the parallel combination of all N_i's will realize the matrix Y.

We now illustrate the results by two examples.

<u>Example 1.</u> It is required to realize the function Y given below as the driving point admittance of a 5-variable network.

$$Y = \frac{2p_2p_5p_1+2p_2p_3p_1+2p_3p_4p_1+p_3p_5p_1 +4p_2p_3p_4+4p_2p_4p_5+2p_3p_4p_5+4p_2p_4p_1}{4p_2p_4+2p_2p_5+2p_2p_3+2p_3p_4+p_3p_5}$$

$$= \frac{V(Y)}{W_{1,2}(Y)}$$

Consider the term $2p_2p_5p_1$. The tree corresponding to this term will consist of the edges e_2, e_5 and e_1. The fundamental cutset matrix C_f of the required network N is given below.

$$C_f = \begin{array}{c} \\ e_2 \\ e_5 \\ e_1 \end{array} \begin{array}{c} \begin{array}{ccccc} e_2 & e_5 & e_1 & e_3 & e_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

For example, the column corresponding to e_3 is obtained as follows.

We first form $(T - e_2) \cup e_3 = (e_5,e_1,e_3)$. There is a term in V(Y) corresponding to (e_5,e_1,e_3) namely, $p_3p_5p_1$. Hence C_{23}, i.e. the entry of C_f at the intersection of the row and column corresponding to the edges e_2 and e_3 respectively, is equal to 1. Since there is a term in V(Y) corresponding to $(T - e_5) \cup e_3 = (e_2,e_3,e_1)$, $C_{53} = 1$. But there exists no term in V(Y) corresponding to $(T - e_1) \cup e_3 = (e_2,e_5,e_3)$. Hence $C_{13} = 0$. The graph G can be constructed. The terminals of edge e_1, form the input-terminal-pair since the variable p_1 is not present in $W_{1,2}(Y)$.

We have to determine the constant K_i's.

Consider e_3. For the tree $T = (e_2,e_3,e_4)$ there exists a 2-tree $(T - e_3) = (e_2,e_4)$. In this case

$$K_a^* = 4p_2p_3p_4$$

and $$K_b^* = 4p_2p_4$$

$$\frac{K_a^*}{K_b^*} = p_3$$

Hence $K_3 = 1$.

Thus all constants K_i's can be determined and are given below:

$$K_1 = 1 \quad K_3 = 1 \quad K_5 = 1$$
$$K_2 = 2 \quad K_4 = 2$$

The network N realizing Y is shown in Fig. 1.

<u>Example 2.</u> It is required to realize the matrix Y given below as the short-circuit admittance matrix of a multivariable network.

$$\begin{bmatrix} \frac{7p_1p_2+6p_1+4p_2}{(p_1+1)(p_2+2)} & \frac{-3p_1p_2-2p_1-2p_2}{(p_1+1)(p_2+2)} & \frac{p_1}{p_1+1} \\ \frac{-3p_1p_2-2p_1-2p_2}{(p_1+1)(p_2+2)} & \frac{6p_1p_2+6p_1+3p_2}{(p_1+1)(p_2+2)} & \frac{p_2}{p_2+2} \\ \frac{p_1}{p_1+1} & \frac{p_2}{(p_2+2)} & \frac{8p_1p_2+4p_2+8p_1}{(p_1+1)(p_2+2)} \end{bmatrix}$$

Y can be decomposed as follows

$$Y = \frac{p_1}{p_1+1} \begin{bmatrix} 3 & -1 & 1 \\ -1 & 3 & 0 \\ 1 & 0 & 4 \end{bmatrix} + \frac{p_2}{p_2+2} \begin{bmatrix} 4 & -2 & 0 \\ -2 & 3 & 1 \\ 0 & 1 & 4 \end{bmatrix}$$

$$= \frac{p_1}{p_1+1} K_1 + \frac{p_2}{p_2+2} K_2$$

It can be noted that K_1 and K_2 are dominant. Hence they can be easily realized by 3-port resistive networks [3], [4], having the same modified cutset matrix.

The 3-port networks N_1^* and N_2^* are shown in Fig. 2. The networks N_1 and N_2 realizing $\frac{p_1}{p_1+1} K_1$ and $\frac{p_2}{p_2+2}$ are shown in Fig. 3. The parallel combination of N_1 and N_2 realizes the matrix Y.

Since a number of equivalent networks can be obtained for N_1^* and N_2^* such that they have the same modified cutset matrix [4] it is possible to obtain a number of equivalent networks realizing Y.

References

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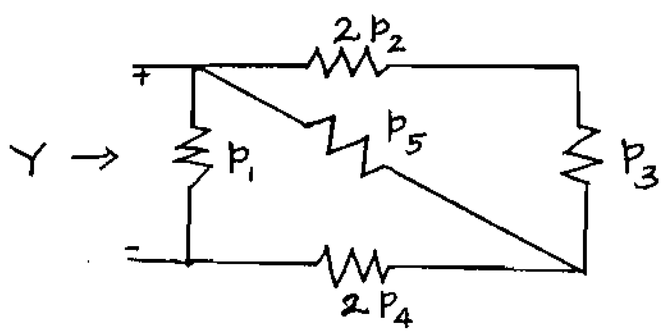
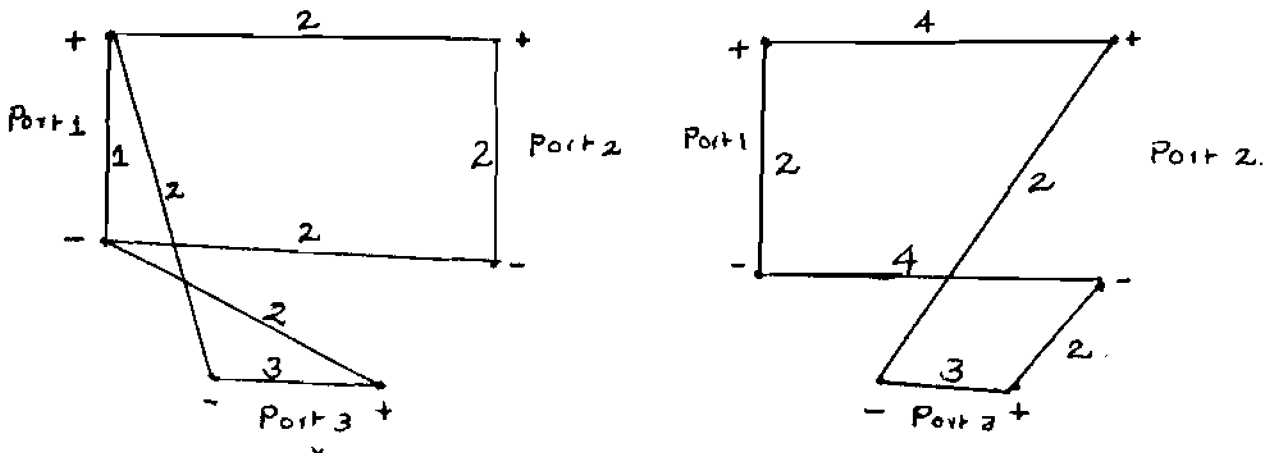


Fig. 1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Port 2
Port 3
N_1^*
Port 1
Port 2.
Port 3
N_2^*

Fig. 2

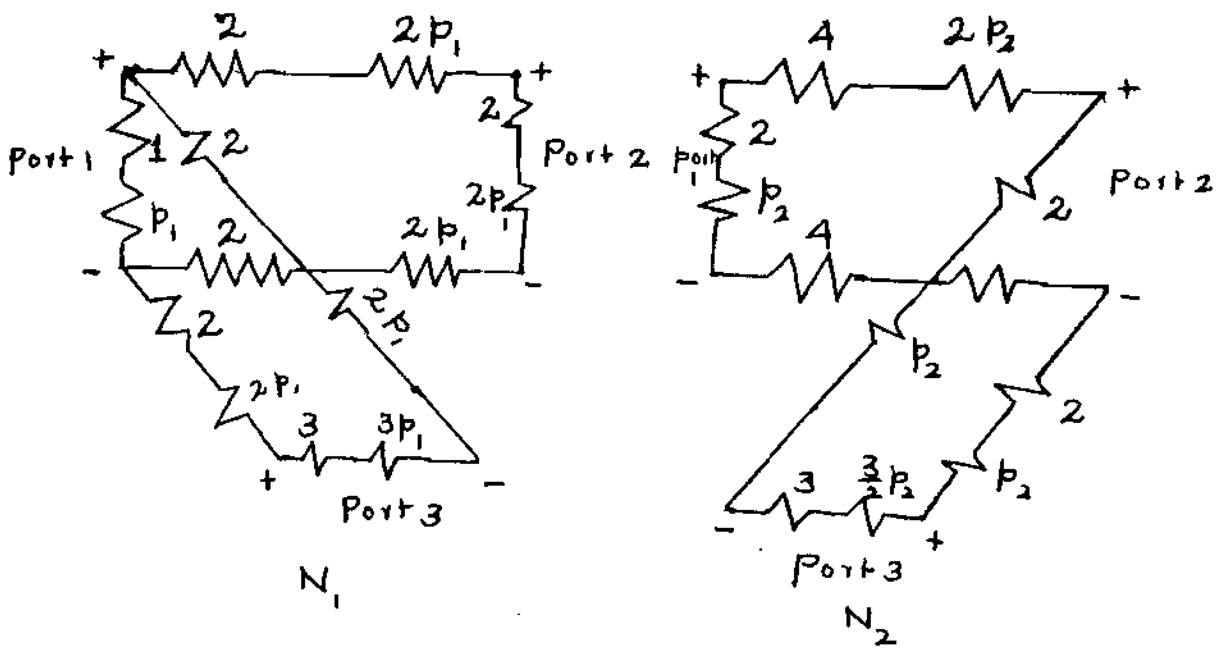
Port 1
Port 2
Port 3
N_1
Port 1
Port 2
Port 3
N_2

Fig. 3.