SYNTHESIS OF THE K-MATRIX OF (n+1)-NODE RESISTIVE n-PORT NETWORKS

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Abstract

In this paper a new procedure for the synthesis of the K-matrix of an (n+1)-node resistive n-port network is given.

1. INTRODUCTION

Recently Lempel and Cederbaum have considered the synthesis of the K-matrices of resistive n-port networks. Synthesis of the K-matrix essentially requires the solution of the following two problems:

(1) Determination of the port configuration T appropriate to the given K-matrix.

(2) Determination of an n-port network containing no negative conductances and having the specified K-matrix.

Lempel and Cederbaum have provided a solution to both the problems. In a companion paper [2] an alternate approach is suggested to solve the second problem using the concepts of padding network and network of departure.

In their solution of the first problem Lempel and Cederbaum first assume the port configuration to be in n parts and obtain the modified cut-set matrix, appropriate to the assumed port configuration and the given K-matrix by making use of a matrix equation relating the modified cut-set matrix and the K-matrix. From the modified cut-set matrix so obtained, the port configuration is determined by the application of a simple procedure. Though this procedure yields a unique port configuration T for a given K-matrix, in some cases, the port configuration T is found, as pointed out later in this paper, to be inconsistent with the matrix. In such cases, the inconsistency will be revealed after a time consuming process of linear programming employed to solve the second problem. Hence, a new procedure for the determination of the port configuration appropriate to a given K-matrix is called for.

Consider a resistive n-port network N having no internal vertices. Let the port configuration T of N be in p parts, T_i, $i = 1,2,\ldots,p$. If no virtual short-circuits are present in N, then the K-matrix of N can be partitioned as

$$K = [k_{ij}] = \begin{bmatrix} K_{11} & K_{12} & \cdots & K_{1p} \\ K_{21} & K_{22} & \cdots & K_{2p} \\ \vdots & \vdots & & \vdots \\ K_{p1} & K_{p2} & \cdots & K_{pp} \end{bmatrix}$$

where

(1) the rows and columns of the submatrix K_{ij}, $i \neq j$ correspond to the ports in T_i and T_j respectively;

(2) each submatrix K_{ii} has entries comprising of 1's and 0's only and is uniquely determined by T_i;

(3) all diagonal entries of K are equal to unity; and

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(4) Each entry in K_{ij}, $j \neq 0$ is less than or equal to unity and greater than or equal to zero, except in degenerat cases.

From the above it may be seen that the problem of determining the port configuration $T = T_1 U T_2 U \ldots T_p$ appropriate to the K-matrix of a résistive n-port network having more than (n+1) nodes is equivalent to the synthesis of the K-matrix of (n+1)-node n-port networks. Hence we consider in this paper the synthesis of the K-matrix of (n+1)-node n-port networks.

II. SYNTHESIS OF THE K-MATRIX OF (n+1)-NODE n-PORT NETWORKS

Consider an (n+1)-node n-port network N. Let T be the port configuration and K, the potential factor matrix of N. It is assumed that each port edge is oriented toward the negative reference terminal. The following theorems can be proved easily using the definition for k_{ij}'s, the potential factors [3].

Theorem 1. Row i of K corresponds to a tip-port if and only if all k_{ij}'s, $j \neq i$ are equal.

Theorem 2. A tip-port is oriented toward (away from) the tip-vertex at which it is incident, if and only if all k_{ij}'s are equal to unity (zero).

Let j and m be any two tip-ports in T. Let the unique path in T containing the ports j and m be denoted by P_{jm}.

Theorem 3. Any non-tip-port x of T will be in P_{jm} if and only if $k_{xj} = 0$ and $k_{xm} = 1$ or vice versa.

Theorem 4. There exists a path containing the terminals of port j and the negative (positive) reference terminal of port i and not containing the positive (negative) reference terminal of port i, if and only if $k_{ij} = 0$ (1).

Based on the above theorems, we now give a procedure for the determination of the port configuration T appropriate to the K-matrix of an (n+1)-node resistive n-port network.

Step 1. Identify all the tip ports of T using Theorem 1.

Step 2. Determine the orientations of the tip-ports using Theorem 2.

Step 3. Let S denote the set of all tip-ports. Let a port j be in S. Determine all P_{jm}'s, $m \in S$, using theorems 3 and 4.

Step 4. Use the paths P_{jm}'s to determine the relative locations of the ports in T and hence T.

Step 5. If the potential factor matrix corresponding to this port configuration T is equal to K, then T is the required port configuration; otherwise K is not realizable.

Example 1. Consider the matrix

$$K = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

This matrix has been used in Example 5 of [2].

It may be observed that in this matrix, in no row all the off-diagonal entries are equal. Hence, by Theorem 1, there exists no row which corresponds to a tip port. However, for any T, there must be at least two tip-ports. Hence the matrix can not be realized as the K-matrix of an (n+1)-node resistive n-port network. This is shown to be true in [2] after solving a linear program.

Example 2. Consider the matrix given by

$$K = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

It is required to determine the port-configuration of the 5-port network for which K is the potential factor matrix.

Step 1. Using Theorem 1, ports 1,3 and 5 are identified as the tip-ports.

Step 2. Ports 1 and 3 are oriented away from the respective tip vertices and port 5 is oriented towards the tip-vertex.

Step 3. If E_{ij} denotes the set of all ports in P_{ij} then using Theorem 3, we can determine E_{13} and E_{15} as follows:

$$E_{13}=\{1,2,3\}, \quad E_{15}=\{1,2,4,5\}$$

Using Theorem 4, P_{13} and P_{15} are determined and are shown in Fig. 1. The required port configuration T is shown in Fig. 2.

The potential factor matrix corresponding to T is seen to be equal to the given matrix K. Hence T is the required port-configuration.

III. CONCLUSIONS

The procedure given by Lempel and Cederbaum to determine the port-configuration corresponding to a given matrix K is unique. However in certain cases, as pointed out in Example 1, the port-configuration may not be consistent with the given K-matrix. This inconsistency will be revealed only after a time consuming process of solving a linear program. The procedure given in this paper enables one to detect the non-realizability of the matrices quite early in the realization procedure thereby obviating the need to solve a linear program.

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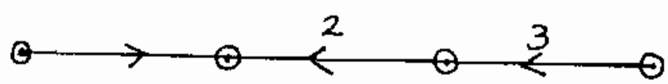
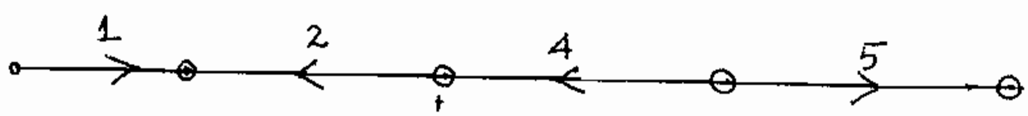


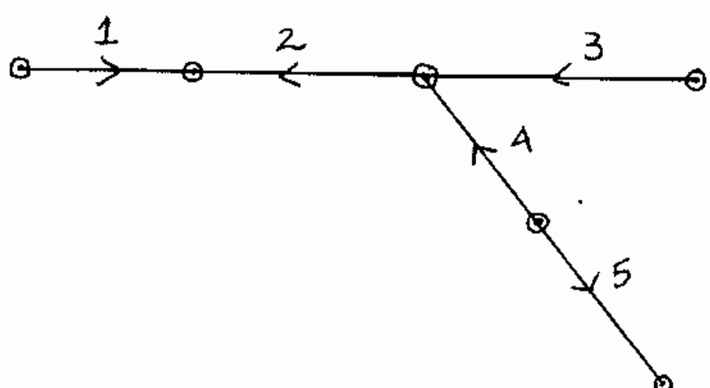
P_{13}

Fig. 1(a)



P_{15}

Fig 1(b)



port configuration T

Fig 2