RELATIONSHIP BETWEEN CUTSET AND NODE-PAIR TRANSFORMATION MATRICES

Indexing terms: Matrix algebra, Network topology

In the letter, a necessary and sufficient condition for a cutset matrix to be a node-pair transformation matrix is established.

In this letter, we answer the following problem posed in Reference 1:

Find the necessary and sufficient conditions that a cutset matrix Q must satisfy if, in

$$V = Q^t V_p$$

the variables in V_p are to be node-pair voltages; i.e. when is a cutset matrix also a node-pair transformation matrix?

Consider a network N. Let G be the linear graph of N. Let the column matrix of edge voltages of G be denoted by V. Let Q be a cutset matrix of G corresponding to a set of $v-1$ independent cutsets, where v is the number of vertexes of G. Then $V = Q^t V_p$. Let row i of Q correspond to cutset q_i.

We assume, without loss of generality, that G contains no parallel edges.

Consider the complete graph G^* constructed on the vertexes of G. Then $G \subseteq G^*$, and the vertex set of G is the same as that of G^*.

Let the vertex set of G be separated into two mutually disjoint subsets A_i and B_i when the edges of q_i are removed from G. Then it is well known that each edge in q_i has one vertex in A_i and the other vertex in B_i. Let the set of edges of G^* which connect vertexes in A_i to vertexes in B_i be denoted by q_i^*; then $q_i \subseteq q_i^*$. Let Q^* be the cutset matrix of G^* corresponding to the cutsets q_i^*, $i = 1, 2, \ldots, v-1$. Since $q_i \subseteq q_i^*$, Q is a submatrix of Q^*. It should be noted that, given Q, Q^* is unique.

Theorem: The variables in V_p are node-pair voltages if, and only if, Q^* (of which Q is a submatrix) is a fundamental cutset matrix of G^* which is a complete graph constructed on the vertexes of G.

Proof of sufficiency: Let Q^* be a fundamental cutset matrix of G^* with respect to a tree T^*. Let the column matrix of voltages of the edges of T^* be denoted by V_t^*. Then

$$V = Q^t V_t^* \qquad (1)$$

The edge voltages of the tree T^* of G^* can be identified with a set of $v-1$ node-pair voltages of G. Hence

$$V = Q^t V_p$$

where the variables in V_p are node-pair voltages.

Proof of necessity: Let the variables in V_p be node-pair voltages of G. Each node-pair voltage of G can be identified with the voltage of an edge of G^*, since G^* is complete. Let the subgraph of such edges of G^* be denoted by G_s^*. G_s^* contains $v-1$ edges, and it contains no circuits, since the node-pair variables in V_p are independent. Hence[1] G_s^* is a tree of G^*. Let the column matrix of voltages of the edges of G_s^* be denoted by V_t^*. Then

$$V = (Q_1^*)^t V_t^* \qquad (2)$$

where Q_1^* is a submatrix of the fundamental cutset matrix Q_f^* of G^* with respect to the tree G_s^* and the columns of Q_1^* correspond to the edges of G.

But

$$V = Q^t V_p$$
$$= Q^t V_t^* \quad \text{since} \quad V_t^* = V_p \qquad (3)$$

Hence

$$Q^t V_t^* = (Q_1^*)^t V_t^* \qquad (4)$$

Since eqn. 4 is valid for all values of V_t^*,

$$Q_1^* = Q$$

Hence Q is a submatrix of the fundamental cutset matrix Q_f^* of G^*. Thus Q is a submatrix of both Q^* and Q_f^*.

Q^* and Q_f^* are cutset matrices of G^* having $v-1$ independent rows. Hence they are related by a nonsingular matrix D as

$$Q^* = [Q|X] = DQ_f^*$$
$$= D[Q|Y] \qquad (5)$$

where X and Y are submatrices of Q^* and Q_f^*, respectively, corresponding to the edges of $(G^* - G)$.

Since the rows of Q are also independent, it follows from eqn. 5 that D is the unit matrix. Hence

$$Q^* = Q_f^*$$

Hence Q^* is a fundamental cutset matrix of G^*.

Thus necessity of the theorem follows.

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Reference

1 SESHU, S., and REED, M. B.: 'Linear graphs and electrical networks' (Addison Wesley, USA), p. 294