Synthesis of Resistive Networks from Third-Order Paramount Matrices

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***Abstract*—A new resistive 3-port network configuration capable of realizing any given third-order real symmetric paramount matrix is given. All the edge conductances are explicitly expressed in terms of the entries of the given matrix obviating matrix inversion at any stage of the calculation of the edge conductance values. Also all kinds of degeneracies are taken care of by the explicit formulas given for the edge conductances.**

I. Introduction

THE PROBLEM OF SYNTHESIS of resistive n-port networks has defied solution for about two decades. A breakthrough in this area of network theory was made by Cederbaum [1], when he proved that paramountcy is a necessary condition for the realization of a real symmetric $(n \times n)$ matrix as the short-circuit conductance or open-circuit resistance matrix of a resistive n-port network. This condition is not sufficient in general. However Tellegen [2] established the sufficiency of this condition for $n \leq 3$ by giving a general network configuration capable of realizing any third-order real symmetric paramount matrix. Since the configuration proposed by Tellegen, is planar, any matrix, realizable as an open-circuit resistance matrix by this configuration, is also realizable as a short-circuit conductance matrix by a network having a dual configuration. Weinberg has reported the details of the synthesis procedure [3] for a short-circuit conductance matrix.

It is well known that a uniformly tapered $(n \times n)$ matrix [4] can be realized as the Y-matrix of an n-port network having a linear-tree port configuration. Cederbaum [5] has shown that any nonuniformly tapered third-order paramount matrix can be realized as the Z-matrix of a 3-port network. To achieve this, he has proved an important property of third-order paramount matrices, namely, the inverse (if it exists) of any nonuniformly tapered third-order paramount matrix is uniformly tapered.

In this paper, we establish a new configuration that can realize any third-order paramount matrix. We also base our realization procedure on a short-circuit conductance matrix.

The results reported in [6] will form the basis of the discussions that follow.

Manuscript received April 11, 1974; revised July 8, 1975.
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	PATTERN Sl. No.	SIGNS y_{12}	y_{13}	y_{23}
GROUP A	1	-	-	-
	2	-	+	+
	3	+	+	-
	4	+	-	+
GROUP B	1	+	+	+
	2	+	-	-
	3	-	-	+
	4	-	+	-

Fig. 1. Various possible sign patterns of matrix Y.

II. A Special Property of a Class of Third-Order Paramount Matrices

Consider a third-order real symmetric paramount matrix Y

$$Y = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{12} & y_{22} & y_{23} \\ y_{13} & y_{23} & y_{33} \end{bmatrix}. \tag{1}$$

All the eight possible sign patterns of the off-diagonal entries of this matrix are given in Fig. 1. In arriving at the sign pattern, the sign corresponding to a zero entry may be arbitrarily chosen. The sign patterns are divided into two groups, A and B. It may be noted that any sign pattern in one group can be obtained from any other sign pattern in the same group by changing the signs of all the entries in an appropriate row and the corresponding column of the matrix. We shall now prove the following theorem.

Theorem 1

In a third-order real symmetric paramount matrix Y, whose off-diagonal entries have a sign pattern belonging to Group A, at least two rows are dominant.

Proof: Let the off-diagonal entries of Y have the 4th-sign pattern in Group A. Let M_{ij} denote a second-order minor of Y obtained by deleting row i and column j. Assume that, in Y, the first row is not dominant. Then we consider the following minors:

$$M_{22} = y_{11}y_{33} - |y_{13}|^2$$

$$M_{21} = y_{12}y_{33} + |y_{13}|y_{23}$$

$$M_{33} = y_{11}y_{22} - y_{12}^2$$

$$M_{31} = y_{12}y_{23} + |y_{13}|y_{22}.$$

Since the matrix Y is paramount and all the above minors are nonnegative we have

$$M_{22} \geq M_{21}$$
$$M_{33} \geq M_{31}.$$

That is,

$$y_{11}y_{33} - |y_{13}|^2 \geq y_{12}y_{33} + |y_{13}|y_{23}$$
$$y_{11}y_{22} - y_{12}^2 \geq y_{12}y_{23} + |y_{13}|y_{22}. \quad (2)$$

Let

$$y_{11} = y_{12} + |y_{13}| + \varepsilon. \quad (3)$$

Substituting (3) in (2), we get

$$(y_{12} + |y_{13}| + \varepsilon)y_{33} - |y_{13}|^2 \geq y_{12}y_{33} + |y_{13}|y_{23}$$
$$(y_{12} + |y_{13}| + \varepsilon)y_{22} - y_{12}^2 \geq y_{12}y_{23} + |y_{13}|y_{22}. \quad (4)$$

We get from (4),

$$(|y_{13}| + \varepsilon)y_{33} \geq |y_{13}|(|y_{13}| + y_{23})$$
$$(y_{12} + \varepsilon)y_{22} \geq y_{12}(y_{23} + y_{12}). \quad (5)$$

Since ε is negative, when the first row is not dominant, for the inequalities in (4) to be true, it is necessary that

$$y_{33} > |y_{13}| + y_{23}$$
$$y_{22} > y_{12} + y_{23}. \quad (6)$$

Thus, we conclude from (6), that if row 1 is not dominant, then 2 and 3 should be dominant, in fact super-dominant. Hence the theorem is true, if the sign pattern corresponds to the 4th one in group A and row 1 is not dominant.

Noting that i) interchanging a row and the corresponding column and/or ii) changing the signs of all the entries in any row and the corresponding column, will affect neither the paramountcy nor the dominance character, we conclude that the theorem is true for any given third-order real symmetric paramount matrix with the sign pattern of the off-diagonal entries belonging to group A.

III. A New Network Configuration Realizing Any Third-Order Paramount Matrix

We establish in this section, a new network configuration realizing any real symmetric paramount matrix of order three as a short-circuit conductance matrix. Let the Y-matrix given in (1) be required to be realized. Let the configuration of the 3-ports of the required network be as shown in Fig. 2. We first determine the unique network of departure N_d [6], [7], having Y as the short-circuit conductance matrix and having the port configuration as shown in Fig. 2. We then determine a set of required parameters to generate a suitable padding 3-port network N_p, such that the parallel combination N of N_d and N_p does not contain any negative conductances. The resulting 3-port network will realize the matrix Y. We follow the notation introduced in [8].

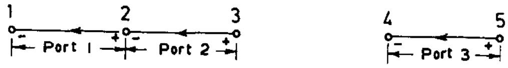

Fig. 2. Port configuration used for realizing given third-order paramount matrix.

Let $G(Y)$ denote the set of all matrices which can be obtained from a given real symmetric third-order paramount matrix by applying to it an arbitrary number of operations of the following two types:

a) changing the signs of all the entries in any row and the corresponding column,
b) interchanging a row and the corresponding column.

Case 1

Let the set $G(Y)$ contain a matrix Y satisfying the following conditions:

i) the off-diagonal entries have the 4th-sign pattern in Group A,
ii) rows 2 and 3 are dominant,
iii) $y_{22}|y_{13}|(y_{33} - y_{23} - |y_{13}|) \geq y_{33}y_{12}(y_{22} - y_{23} - y_{12}) \geq 0.$

The reason for the condition iii) above, will become clear as we proceed further in our discussions.

Since the operation a) is equivalent to changing the orientations of the ports and b) equivalent to renumbering the ports, the conditions stated above lead to no loss of generality.

To begin with we assume that all y_{ij}'s are nonzero and derive expressions for the edge conductances g_{ij}'s, in terms of y_{ij}'s. These expressions, will then be used to derive formulas for g_{ij}'s, for the case of a Y-matrix having one of its y_{ij}'s equal to zero. We shall treat as trivial, matrices with two zero entries, since realizations in such cases will be in two connected components and can be easily obtained.

The edge conductances of the network of departure N_d, having Y as its short-circuit conductance matrix and having a port configuration as shown in Fig. 2, are obtained as

$$(g_{12})_d = y_{11} - y_{12}$$
$$(g_{13})_d = y_{12}$$
$$(g_{23})_d = y_{22} - y_{12}$$
$$(g_{14})_d = |y_{13}|$$
$$(g_{15})_d = -|y_{13}|$$
$$(g_{24})_d = -(|y_{13}| + y_{23})$$
$$(g_{25})_d = (|y_{13}| + y_{23})$$
$$(g_{34})_d = y_{23}$$
$$(g_{35})_d = -y_{23}$$
$$(g_{45})_d = y_{33}. \quad (7)$$

The edge conductances of a suitable padding 3-port network are to be obtained, from the following expressions derived in [6]:

$$(g_{ij})_p = \frac{-S_iS_j}{S}, \qquad i,j = 1,2,3\ (i \neq j) \text{ or } i,j = 4,5\ (i \neq j)$$

$$(g_{ij})_p = \frac{S_iS_j}{S}, \qquad i = 1,2,3;\ \ j = 4,5 \tag{8}$$

where

$$S_1 + S_2 + S_3 = S_4 + S_5 = S. \tag{9}$$

Let

$$\frac{S_i}{\sqrt{S}} = s_i.$$

Equations (8) become

$$(g_{ij})_p = -s_is_j, \qquad i,j = 1,2,3\ (i \neq j) \text{ or } i,j = 4,5\ (i \neq j)$$

$$(g_{ij})_p = s_is_j, \qquad i = 1,2,3;\ \ j = 4,5. \tag{10}$$

We have to choose suitable values for s_i's such that

$$g_{ij} = (g_{ij})_d + (g_{ij})_p \geq 0, \qquad \text{for all } i,j = 1,2,3,4,5\ (i \neq j).$$

Let us choose s_i's such that the edge conductances g_{13}, g_{15}, g_{23}, and g_{24} are zero. That is,

$$\begin{aligned}(g_{13})_d + (g_{13})_p &= 0\\ (g_{15})_d + (g_{15})_p &= 0\\ (g_{23})_d + (g_{23})_p &= 0\\ (g_{24})_d + (g_{24})_p &= 0.\end{aligned} \tag{11}$$

On an examination of (7) and (10), it is clear that the corresponding edge conductances of N_d and N_p appearing in each one of (11) are of opposite sign and so, we are justified in attempting to choose nonnegative s_i's using (11). Substituting (7) and (10) in (11), we have

$$\begin{aligned}s_1s_3 &= y_{12}\\ s_1s_5 &= |y_{13}|\\ s_2s_3 &= y_{22} - y_{12}\\ s_2s_4 &= |y_{13}| + y_{23}.\end{aligned} \tag{12}$$

We solve (12) and express s_2, s_3, s_4, and s_5 in terms of s_1 as follows:

$$\begin{aligned}s_3 &= \frac{y_{12}}{s_1}\\ s_5 &= \frac{|y_{13}|}{s_1}\\ s_2 &= \frac{y_{22} - y_{12}}{y_{12}}\, s_1\\ s_4 &= \frac{(|y_{13}| + y_{23})y_{12}}{(y_{22} - y_{12})s_1}.\end{aligned} \tag{13}$$

Substituting (13) in (9), we get

$$s_1^2 = \frac{y_{12}(y_{12}y_{23} + |y_{13}|y_{22} - y_{12}y_{22} + y_{12}^2)}{y_{22}(y_{22} - y_{12})}. \tag{14}$$

Using (13) and the value of s_1^2 as given in (14), let us next check, if the remaining nonzero edge conductances g_{ij}'s obtained as the algebraic sum of the corresponding $(g_{ij})_d$'s and $(g_{ij})_p$'s are nonnegative.

We get from (7) and (10) and the equation

$$g_{ij} = (g_{ij})_d + (g_{ij})_p$$

the following equations:

$$\begin{aligned}g_{14} &= |y_{13}| + s_1s_4\\ g_{25} &= (|y_{13}| + y_{23}) + s_2s_5\\ g_{34} &= y_{23} + s_3s_4\\ g_{12} &= (y_{11} - y_{12}) - s_1s_2\\ g_{45} &= y_{33} - s_4s_5\\ g_{35} &= -y_{23} + s_3s_5.\end{aligned} \tag{15}$$

Substituting (13) and (14) in (15), we get

$$g_{14} = |y_{13}| + \frac{(|y_{13}| + y_{23})y_{12}}{(y_{22} - y_{12})} \tag{16a}$$

$$g_{25} = (|y_{13}| + y_{23}) + \frac{(y_{22} - y_{12})|y_{13}|}{y_{12}} \tag{16b}$$

$$g_{34} = y_{23} + \frac{y_{12}y_{22}(|y_{13}| + y_{23})}{(y_{12}y_{23} + |y_{13}|y_{22} - y_{12}y_{22} + y_{12}^2)} \tag{16c}$$

$$g_{12} = \frac{(y_{11}y_{22} - y_{12}^2) - (y_{12}y_{23} + |y_{13}|y_{22})}{y_{22}} \tag{17a}$$

$$g_{45} = \frac{y_{22}|y_{13}|(y_{33} - y_{23} - |y_{13}|) - y_{33}y_{12}(y_{22} - y_{23} - y_{12})}{(y_{12}y_{23} + |y_{13}|y_{22} - y_{12}y_{22} + y_{12}^2)} \tag{17b}$$

$$g_{35} = \frac{(y_{22} - y_{23} - y_{12})(|y_{13}|y_{22} + y_{12}y_{23})}{(y_{12}y_{23} + |y_{13}|y_{22} - y_{12}y_{22} + y_{12}^2)}. \tag{17c}$$

Note that the inequality used in condition iii) may be rewritten as

$$y_{33}(y_{12}y_{23} + |y_{13}|y_{22} - y_{12}y_{22} + y_{12}^2) \geq y_{22}|y_{13}|y_{23} + y_{22}|y_{13}|^2 \geq 0. \tag{18}$$

It can be seen from the above that

$$\Delta_1 = (y_{12}y_{23} + |y_{13}|y_{22} - y_{12}y_{22} + y_{12}^2) > 0$$

if all y_{ij}'s are nonzero.

Further by assumption, row 2 is dominant. Hence $y_{22} \neq y_{12}$ if all y_{ij}'s are nonzero.

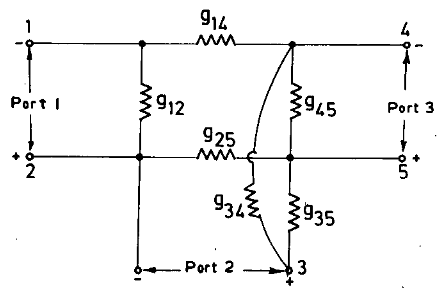

Fig. 3. General network configuration realizing any third-order paramount Y-matrix.

The above observations together with the paramountcy of Y and conditions ii) and iii) prove that the expressions given in (16) and (17) are meaningful and the corresponding g_{ij}'s are nonnegative.

The steps for realizing a third-order real symmetric paramount conductance matrix having no zero entries and satisfying the three conditions, may be stated as follows.

Step 1

Check, if the given matrix satisfies the following conditions:

i) the off-diagonal entries have the 4th-sign pattern in Group A,
ii) rows 2 and 3 are dominant,
iii) $y_{22}|y_{13}|(y_{33} - y_{23} - |y_{13}|) \geq y_{33}y_{12}(y_{22} - y_{23} - y_{12}) \geq 0.$

If not, obtain a matrix Y, by performing one or more of the following operations on the given matrix:

a) changing the signs of all the entries in any row and the corresponding column,
b) interchanging a row and the corresponding column.

Step 2

Calculate the conductances g_{14}, g_{25}, g_{34}, g_{12}, g_{45}, and g_{35} using (16) and (17).

Step 3

Set the other conductances to zero.

Step 4

The 3-port network realizing the matrix Y, as a short-circuit conductance matrix will be as shown in Fig. 3.

Step 5

By suitably renumbering the ports, and changing their orientations (necessitated because of the operations performed in Step 1) the 3-port network, realizing the given matrix can be obtained.

Matrices with one of the y_{ij}'s equal to *zero* can be handled as follows.

Case 1(a): $y_{12} = 0$

In this case it can be seen from (18) that $\Delta_1 > 0$. The conductance g_{25} will be a short-circuit and all other conductances can be obtained using (16) and (17).

Case 1(b): $y_{23} = 0$

Again it can be seen that $\Delta_1 > 0$. The expressions for g_{ij}'s given by (16) and (17) can be used after substituting in them $y_{23} = 0$.

However, if in addition $y_{22} = y_{12}$, then g_{14} will be a short-circuit.

Case 1(c): $y_{13} = 0$

In this case condition iii) will be satisfied only if

$$y_{22} = y_{23} + y_{12}$$

but then $\Delta_1 = 0$.

To handle this case one can interchange rows 1 and 2 and the corresponding columns so that in the resulting matrix the zero entry will appear in the (2,3) position. Then any necessary row and column interchanges as well as row and column sign changes may be performed so that the matrices resulting from such changes will satisfy conditions i)–iii). The discussions of Case 1(b) will then apply to this new matrix.

Case 2

Let the set $G(Y)$ contain a matrix Y satisfying the following conditions:

i) the off-diagonal entries have the 2nd sign-pattern in Group B,
ii) $|y_{23}| \leq \min\{y_{12}, |y_{13}|\}$,
iii) $y_{22}|y_{13}|(y_{33} + |y_{23}| - |y_{13}|) \geq y_{33}y_{12}(y_{22} + |y_{23}| - y_{12}) \geq 0.$

To begin with we assume that all y_{ij}'s are nonzero and $y_{13} \neq y_{23}$.

Let the same port configuration given in Fig. 2, be assumed to construct the network of departure with respect to the Y-matrix. All the edge conductances of the network of departure N_d with respect to Y will be obtained, from (7), by replacing y_{23} by $-|y_{23}|$. Here again, we try to generate a 3-port padding network N_p, such that g_{13}, g_{15}, g_{23}, and g_{24} are zero. We follow the same procedure as in Case 1 and obtain expressions for the nonzero edge conductances in terms of the Y-matrix entries as follows:

$$g_{14} = |y_{13}| + \frac{(|y_{13}| - |y_{23}|)y_{12}}{(y_{22} - y_{12})} \tag{19a}$$

$$g_{25} = (|y_{13}| - |y_{23}|) + \frac{(y_{22} - y_{12})|y_{13}|}{y_{12}} \tag{19b}$$

$$g_{35} = |y_{23}| + \frac{|y_{13}|y_{22}(y_{22} - y_{12})}{(-y_{12}|y_{23}| + |y_{13}|y_{22} - y_{12}y_{22} + y_{12}^2)} \tag{19c}$$

$$g_{12} = \frac{(y_{11}y_{22} - y_{12}^2) - (-y_{12}|y_{23}| + |y_{13}|y_{22})}{y_{22}} \tag{20a}$$

$$g_{45} = \frac{y_{22}|y_{13}|(y_{33} + |y_{23}| - |y_{13}) - y_{33}y_{12}(y_{22} + |y_{23}| - y_{12})}{(-y_{12}|y_{23}| + |y_{13}|y_{22} - y_{12}y_{22} + y_{12}^2)} \tag{20b}$$

$$g_{34} = \frac{(y_{12} - |y_{23}|)(y_{22}|y_{13}| - |y_{23}|y_{12})}{(-y_{12}|y_{23}| + |y_{13}|y_{22} - y_{12}y_{22} + y_{12}^2)}. \tag{20c}$$

Rearranging the inequality of condition iii) we obtain

$$y_{33}(-y_{12}|y_{23}| + |y_{13}|y_{22} - y_{12}y_{22} + y_{12}^2) \geq y_{22}|y_{13}|(|y_{13}| - |y_{23}|) \geq 0. \tag{21}$$

It can be seen from the above that

$$\Delta_2 = (-y_{12}|y_{23}| + |y_{13}|y_{22} - y_{12}y_{22} + y_{12}^2) > 0$$

if all y_{ij}'s are nonzero and $y_{13} \neq y_{23}$.

The above observations together with the paramountcy of Y and conditions i)–iii) prove that the expressions (19) and (20) for g_{ij}'s are nonnegative.

The following steps may then be followed to realize a given Y-matrix in which all y_{ij}'s are nonzero and $y_{13} \neq y_{23}$.

Step 1

Check if the given matrix satisfies the following three conditions:

i) the off-diagonal entries have the 2nd-sign pattern in Group B,
ii) $|y_{23}| \leq \min\{y_{12}, |y_{13}|\}$,
iii) $y_{22}|y_{13}|(y_{33} + |y_{23}| - |y_{13}|) \geq y_{33}y_{12}(y_{22} + |y_{23}| - y_{12}) \geq 0$.

If not, obtain a matrix Y, by performing one or more of the following operations on the given matrix:

a) changing the signs of all the entries in any row and the corresponding column,
b) interchanging a row and the corresponding column.

Step 2

Calculate the conductances g_{14}, g_{25}, g_{35}, g_{12}, g_{45}, and g_{34}, using (19) and (20).

Step 3

Set the other conductances to zero.

Step 4

The 3-port network realizing the matrix Y as a short-circuit conductance matrix will be as shown in Fig. 3.

Step 5

By suitably renumbering the ports, and changing their orientations (necessitated because of the operations performed in Step 1), the 3-port network realizing the given matrix can be obtained.

As regards matrices with one of the y_{ij}'s equal to zero, we proceed as follows.

Case 2(a): $y_{23} = 0$

Any y_{ij} equal to zero will appear only in the (2,3) position because of condition ii).

If y_{23} is equal to zero, then it can be seen from (21) that $\Delta_2 > 0$. Thus the expressions for g_{ij}'s given by (19) and (20) will be nonnegative.

If $y_{22} = y_{12}$ in addtion to y_{23} being zero, then g_{14} will be a short-circuit.

Case 2(b): $y_{23} = y_{13}$; $y_{12} > |y_{13}|$

In this case condition iii) will be satisfied only if $y_{22} = y_{12}$. But then $\Delta_2 = 0$. However, by interchanging rows 2 and 3 and the corresponding columns and then changing the signs of entries in row 1 and column 1 of the resulting matrix, one can get a new matrix Y' in which

$$y_{12}' = |y_{23}'|$$

and

$$|y_{13}'| > |y_{23}'|.$$

It can be seen that these y_{ij}'s will satisfy condition iii) and

$$-y_{12}'|y_{23}'| + |y_{13}'|y_{22}' - y_{12}'y_{22}' + (y_{12}')^2 > 0.$$

Thus the (19) and (20) will yield nonnegative g_{ij}'s. Again the conductance g_{14} will be a short-circuit if

$$y_{22}' = y_{12}'.$$

Case 2(c): $y_{12} = |y_{13}| = |y_{23}|$

For this case $\Delta_2 = 0$. Hence g_{35} calculated using (19c) will be a short-circuit and g_{45} and g_{34} will be of the indeterminate form 0/0. But it can be shown that

$$g_{45} + g_{34} = y_{33} - |y_{13}|.$$

Thus the following formulas will yield the g_{ij}'s of the required 3-port network:

$$\begin{aligned} g_{14} &= |y_{13}| \\ g_{25} &= y_{22} - y_{12} \\ g_{35} &= \infty \\ g_{12} &= y_{11} - y_{12} \\ g_{45} + g_{34} &= y_{33} - y_{12}. \end{aligned} \tag{22}$$

We may note that when $|y_{23}| = |y_{13}| = y_{12}$, the matrix Y can be transformed into a uniformly tapered one. The network obtained using the conductances of (22) is the same as the 4-node realization that one will obtain using Guillemin's approach for uniformly tapered matrices [4].

Case 2(d): $y_{22} = y_{12} = |y_{13}| = |y_{23}|$

In this case formulas (22) are applicable with $y_{22} = y_{12}$.

Example 1

Let it be required to realize the following third-order paramount matrix Y as the short-circuit conductance matrix of a 3-port network:

$$Y = \begin{bmatrix} 15 & 0 & -15 \\ 0 & 20 & 10 \\ -15 & 10 & 30 \end{bmatrix}.$$

Taking the sign of the zero entries in the above matrix as positive, we notice that the sign pattern of the off-diagonal entries of Y corresponds to that of Case 1 and it satisfies conditions ii) and iii) also. So, we calculate all the edge conductances (all in mhos) using (16) and (17). The edge conductances of the 3-port network shown in Fig. 3, which realizes the above Y-matrix are as follows:

$$g_{14} = 15$$
$$g_{25} = \infty$$
$$g_{34} = 10$$
$$g_{12} = 0$$
$$g_{45} = 5$$

and

$$g_{35} = 10.$$

Example 2

Let it be required to realize the matrix Y, given by

$$Y = \begin{bmatrix} 20 & 10 & -5 \\ 10 & 10 & -4 \\ -5 & -4 & 8 \end{bmatrix}.$$

We notice that the sign pattern of the off-diagonal entries in the above matrix corresponds to that of Case 2, and it satisfies the conditions ii) and iii) also. The edge conductances (all in mhos) of the 3-port network shown in Fig. 3, realizing the above Y-matrix are obtained by using (19) and (20) and are as follows:

$$g_{14} = \infty$$
$$g_{25} = 1$$
$$g_{35} = 4$$
$$g_{12} = 9$$
$$g_{45} = 3$$

and

$$g_{34} = 6.$$

IV. Conclusions

In this paper, we have given a new general network configuration (Fig. 3), capable of realizing any real symmetric third-order paramount matrix as the short-circuit conductance matrix of a resistive 3-port network. Like Tellegen's configuration, it also contains only 6 elements. All the edge conductances are explicitly expressed in terms of the Y-matrix entries. Also, all cases of degeneracies will be taken care of by the explicit formulas given in the paper for the edge conductances and no matrix inversion is required at any stage [3]. It may be noted that the network configuration in Fig. 3 is planar and so its dual will have the given Y-matrix as its open-circuit resistance matrix. So, if any real symmetric third-order paramount matrix is to be realized as an open-circuit resistance matrix of a resistive 3-port network, we can realize it as the short-circuit conductance matrix and take its dual. Thus the procedure given in this paper can be used to realize third-order paramount Z or Y matrices of 3-port networks.

It may be mentioned that the choice of the network shown in Fig. 3 for realizing paramount matrices has been motivated by the work of Biorci and Civalleri [9] who have presented $(n+2)$-node n-port configurations the conductances of which can be expressed in terms of y_{ij}'s (this is known as "inversion" of the n-port).

Finally, we would like to add that the results of the paper have provided guidelines for certain important results on the $(n+2)$-node realizability of Y-matrices [10].

Acknowledgment

The authors thank the reviewers for their helpful suggestions and constructive criticisms.

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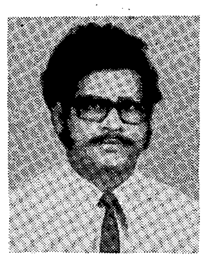

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On the Frequency Weighted Least-Square Design of Finite Duration Filters

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Abstract—**The frequency weighted least-square design of finite-duration filters, in one and two dimensions, continuous and discrete is considered. Some new theoretical results and some practical design techniques for conventional and unconventional filters are presented. In some cases optimum discrete filters can be found conveniently by matrix inversion. In many cases a simple, iterative approximation technique using FFT can be used to carry out the design or to adjust the frequency response of filters.**

I. Introduction

IN THE DESIGN OF finite-duration filters which approximate some desired frequency response, the designer has generally to contend with different approximation requirements for different regions of the frequency spectrum. For one-dimensional sampled data filters with periodic sampling (finite-duration impulse response (FIR) digital filters), a number of design techniques have been developed which achieve these design objectives (see [25]–[27] for excellent summaries). A common and simple technique using data windows approaches the problem in a two step fashion. A trigonometric approximation of the desired frequency response is first obtained. This approximation is optimum in the least-square sense. Then one of several finite duration data windows with good frequency concentration properties is used to modify the previous design and to reduce the unwanted effects of the Gibbs phenomenon in parts of the frequency range. One specially important data window has been derived analytically in the fundamental work of Slepian and Pollak who established the optimal frequency concentration property of one of the prolate spheroidal functions of zero order [1]. The related Kaiser window [2] is commonly and successfully used in the design of FIR digital filters [3]. We consider in this paper an alternate analytic approach to the design of one- and two-dimensional filters and windows of finite duration. This approach differs in philosophy and in detail from previous work in the sense that primary emphasis is given to the constraint of a finite duration and only secondarily to the additional conditions of a continuous or discrete

Manuscript received April 5, 1974; revised July 15, 1974, and June 20, 1975. This work was supported in part by NSF Grant GK-37282 and in part by the National Aeronautic and Space Administration under NASA Grant NGL 05-003-404.

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