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On the Capacity of Hybrid FSO/RF Links

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***Abstract*— Hybrid Free Space Optics (FSO)/Radio Frequency (RF) communication systems have emerged as a way to improve network performance by providing enhanced availability and reliability. In an effort to mitigate individual drawbacks in the optical link during adverse weather conditions, network traffic flows simultaneously between channels. Based on the *Shannon–Hartley* theorem, channel capacity is dependent, among others, on both the signal to noise ratio (SNR) and channel bandwidth. As such, combined link throughput may be affected by channel state conditions. Because atmosphere turbulence can be modeled as a time-varying fading channel, capacity analysis can be investigated. In this paper, authors apply various availability scenarios of channel state information (CSI) on the optical link to derive closed-form expressions for combined link capacity. Numerical simulation provides a comparison to the results.**

***Keywords*— Hybrid FSO/RF link, channel capacity, channel state information (CSI), reconfiguration, and lognormal fading.**

I. Introduction

In the event that an FSO link is obscured in a hybrid FSO/RF ad hoc network, the system switches to a reliable RF to maintain communication. We refer to this scheme as *reconfiguration* [1], and is characterized when SNR decreases below a preset threshold. Accordingly, switching can be accomplished by rerouting traffic via an alternate optical link. Since we assume only one optical link in the considered ad hoc network, the latter case is beyond the scope of this paper. Instead, the RF link is employed to maintain the traffic flow. The capacity of a hybrid link as the upper bound of the limiting information rate is highly dependent on channel condition. A higher capacity exhibits a more reliable transmission link. In this paper, authors model the channel as a time-varying fading channel and then address link capacity. Goldsmith and Varaiya [2] previously evaluated the effect of fading on wireless channel capacity. Basic definitions in [2] are used, and then the authors' assumptions are used to find closed-form capacity expressions for a hybrid link. Non-random analysis of the channel, e.g. scattering, will not be considered. Likewise, routing and traffic engineering—a challenge in hybrid systems—are not discussed. Details, however, can be found in [3-4]. The proposed analysis does not include error control coding, which may be of interest when analyzing channel capacity.

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II. Channel Model

We assume FSO links using intensity modulation and direct detection (IM/DD) with on/off keying (OOK). For an FSO channel with fading channel coefficient h and additive Gaussian noise, the *instantaneous* SNR will be defined by

$$\gamma \triangleq \frac{4h^2 R^2 P_t^2}{(\sigma_1+\sigma_0)^2} \tag{1}$$

where σ_1 and σ_0 are the standard deviation of the noise currents for bits '1' and '0', R is receiver's responsivity and P_t is the average transmitted power. The *average* received SNR can be defined by

$$\bar{\gamma} \triangleq \frac{4R^2 P_t^2}{(\sigma_1+\sigma_0)^2} \tag{2}$$

A. Reconfiguration Model

In FSO communications, fading process has a coherence time τ_c in the order of milliseconds, which is slow when compared to typical symbol rates of FSO systems. If T_b is considered as the symbol period, a value of $T_b/\tau_c \le 0.001$ is applicable in FSO application simulations. Thus, the channel coefficient h and the instantaneous SNR γ can be assumed constant for a large number of transmitted bits. Hence, link outage will occur when the instantaneously received SNR falls below a given threshold, γ_L. Thus reconfiguration is required:

$$\frac{4h^2 R^2 P_t^2}{(\sigma_1+\sigma_0)^2} < \gamma_L \tag{3}$$

The channel coefficient h is the ratio of the faded light intensity I to the intensity without turbulence I_0. This is referred to as the Channel State Information (CSI) and is typically estimated by a channel estimation scheme available through analysis. If h is known, we can conclude from (1) that information on γ is also available.

In the channel model, each state corresponds to a specific channel quality [5]. An example of the effect of the received SNR on the reconfiguration process is shown in Fig. 1. The FSO link functions as long as the received SNR is more than γ_H. The link requires reconfiguration to an RF link after SNR falls below γ_L, shown at the time t_0. To prevent a flapping effect, the hysteresis gap shows the gap between γ_L and γ_H. In this case, the status of the link depends on its previous state. Additional analysis regarding reconfiguration statistics is available in [6]. The channel model can be defined as a three-state Markov chain at any time n

(4)

$$X_n = \begin{cases} S_0 & \text{Avtice status of FSO channel at time } n \\ S_1 & \text{Hystersis gap} \\ S_2 & \text{Reconfiguration to RF link} \end{cases}$$

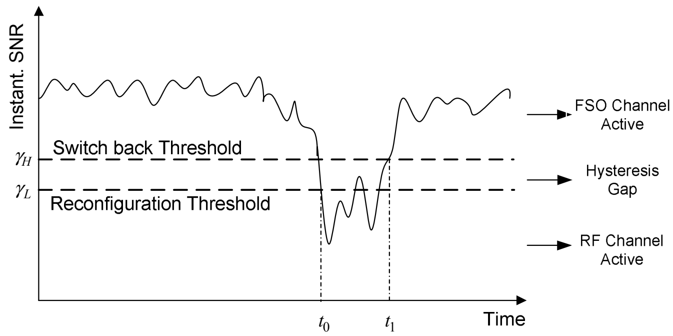

Fig. 1: Reconfiguration models versus instantaneous SNR for an FSO time-varying channel.

B. Fading Models

In wireless optical communications, several fading models are considered for modeling atmosphere-induced turbulence. A *lognormal* distribution of the fading channel has the probability distribution function (PDF) in the form of

$$f_I(h) = \frac{1}{\sqrt{8\pi} h \sigma_x} \exp\left(-\frac{\left[\ln(h) + 2\sigma_x^2\right]^2}{8\sigma_x^2} \right) \quad (5)$$

where σ_x^2 is the variance of log-amplitude fading [7], in this paper referred to as *fading strength*. The fading coefficient h is chosen to normalize $E[h]$ and will be referred to as the normalized irradiance. Recalling a lognormal distribution for normalized irradiance h by (5), the probability of a link being either obscured or too noisy is expressed in terms of stationary probabilities and can be rewritten by [6]

$$\pi_n(2) = \int_0^{\gamma_L} f_I(h)dh = \frac{1}{2} + \frac{1}{2} erf\left[\frac{1}{\sqrt{8}\sigma_x} \left(\ln\left(\sqrt{\frac{\gamma_L}{\bar{\gamma}}} \right) + 2\sigma_x^2 \right) \right] \quad (6)$$

where *erf*(.) is the *error function*. For simplicity, as considered in (6), we may use notation "2" as the representation of S_2 in equations. Similarly, the probability $\pi_n(0)$ as the probability of recovering from an obscured link or in terms of SNR is equal to

$$\pi_n(0) = \int_{\gamma_H}^{\infty} f_I(h)dh = \frac{1}{2} - \frac{1}{2} erf\left[\frac{1}{\sqrt{8}\sigma_x} \left(\ln\left(\sqrt{\frac{\gamma_H}{\bar{\gamma}}} \right) + 2\sigma_x^2 \right) \right] \quad (7)$$

Finally, the probability of being at state 1 can be easily calculated by

$$\pi_n(1) = 1 - \pi_n(0) - \pi_n(2) \quad (8)$$

where $\pi_n(0)$ and $\pi_n(2)$ are given by (7) and (6), respectively.

A *Gamma–Gamma* fading model also provides acceptable matching between model and evaluated data resulting from turbulence induced fading channel. The fading model has a PDF expressed in the form of [8]:

$$f_I(h) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} h^{(\alpha+\beta)/2-1} K_{\alpha-\beta}\left(2\sqrt{\alpha\beta h}\right) \; ; \; h > 0 \quad (9)$$

where $K_v(.)$ is the modified Bessel function of the second kind of order v, $\Gamma(.)$ is the gamma function, and parameters α and β can be directly related to atmospheric conditions. Corresponding expressions can be found in [8]. The probability of an obscured link or excessive noise at time n is expressed in terms of stationary probabilities. Using [6, Eq. (14)], the closed-form expression of reconfiguration probability will be given by

$$\pi_n(2) = 1 - \frac{\alpha\beta}{\Gamma(\alpha)\Gamma(\beta)} \left(\alpha\beta \sqrt{\frac{\gamma_L}{\bar{\gamma}}} \right)^{\frac{\alpha-\beta}{2}} G_{1,3}^{3,0}\left[\alpha\beta \sqrt{\frac{\gamma_L}{\bar{\gamma}}} \middle|_{1-\frac{\alpha+\beta}{2}, \frac{\alpha-\beta}{2}, \frac{\beta-\alpha}{2}}^{-\frac{\alpha+\beta}{2}} \right] \quad (10)$$

Meijer's G-function was used in (10) where $K_v(x) = \frac{1}{2} G_{0,2}^{2,0}\left[x^2/2 \middle|_{v/2,\, -v/2}^{-} \right]$. The stationary probability $\pi_n(0)$ representing the normal operation of FSO link, i.e. no need for reconfiguration, can be similarly calculated by

$$\pi_n(0) = \frac{\alpha\beta}{\Gamma(\alpha)\Gamma(\beta)} \left(\alpha\beta \sqrt{\frac{\gamma_H}{\bar{\gamma}}} \right)^{\frac{\alpha-\beta}{2}} G_{1,3}^{3,0}\left[\alpha\beta \sqrt{\frac{\gamma_H}{\bar{\gamma}}} \middle|_{1-\frac{\alpha+\beta}{2}, \frac{\alpha-\beta}{2}, \frac{\beta-\alpha}{2}}^{-\frac{\alpha+\beta}{2}} \right] \quad (11)$$

MATLAB, which was used for our computations, does not support the Meijer's-G function. The authors used the original integral forms of such probabilities in (6) and (7) to numerically compute the integral.

III. CAPACITY MODEL

The capacity of FSO-based ad hoc mesh networks has received extensive attention because of the high capacity of FSO links. However, the link severe sensitivity to atmospheric condition is a limiting factor. An upper limit for per-node throughput capacity of ad hoc hybrid RF/FSO systems can be found in [9]. The authors in [10] analyzed the throughput benefit of a hybrid network formed by placing a sparse network of base stations as infrastructure in an ad hoc network. However, analyses were based on routing algorithm, node distribution, and transmission strategies. However, this scheme does not address hybrid channel capacity. The proposed study considers the Shannon-based link capacity. The authors assume a single hybrid FSO/RF link between two homogeneous nodes in an ad hoc network equipped with optical and RF transceiver apertures (so called super nodes [9]).

A. Hybrid FSO/RF Model

The RF link, which is assumed always available, is deployed as a redundancy for sensitive FSO links. Three approaches for utilizing a hybrid RF link in conjunction with an FSO link are available. One is based on data duplication on the RF providing data recovery and is useful if a spatial diversity mechanism is used. However, the remarkable difference between radio and optical data rates makes diversity not applicable, unless an adaptive rate algorithm is used on the FSO channel [7]. One may consider the case when an individual RF link may yield the same transmission rate as FSO link. In [11], a combined FSO with E-band RF link is considered so that the capacity has been numerically evaluated, mostly assuming nonrandom channels.

In a second approach user data is sent over a hybridized link in which FSO and RF links dynamically interact with each other to achieve a combined transmission link. Maximum reliability and throughput for hybrid channels is achieved in terms of outage probability [12] and comes with more bandwidth utilization but, of course, higher transceiver design complexity.

A third approach, namely using an RF link as a backup, is assumed in this paper. User data is not transferred on the RF link while the FSO link is in active status. Thus, RF is used primarily for control signals and data transfer whenever FSO is unavailable [1]. However, lower bandwidth is utilized with such simple transceiver design.

B. Capacity of FSO link with Perfect CSI at the Receiver Side

If CSI is available at the receiver side during clear weather conditions, the nominal averaged capacity (bits/sec) of a given FSO link is calculated by integration on the received instantaneous electrical SNR γ [2]

$$C_{fso} = \int_0^\infty B_{fso} \log_2(1+\gamma) f_I(\gamma) d\gamma \qquad (12)$$

where B_{fso} is the FSO channel's bandwidth and $f_x(\gamma)$ is the fading PDF in SNR representation. For a lognormal fading model, when substituting h from (1) and (2) as $h = \sqrt{\gamma / \bar{\gamma}}$ into (5):

$$f_I(\gamma) = \frac{1}{\sqrt{32\pi}\gamma\sigma_x} \exp\left(- \frac{\left[\ln(\gamma/\bar{\gamma}) + 4\sigma_x^2\right]^2}{32\sigma_x^2} \right) \qquad (13)$$

for any $\gamma \geq 0$. Using similar analysis from [13], a closed form expression can then be approximately derived for $C_{fso,L}$ as

$$\begin{aligned} C_{fso,L} \approx & \frac{B_{fso}}{2\ln(2)} \exp\left(- \frac{\left[4\sigma_x^2 - \ln(\bar{\gamma})\right]^2}{32\sigma_x^2} \right) \\ & \times \left[\sum_{k=1}^{K} \frac{(-1)^{k+1}}{k} erfcx\left(\sqrt{2}\sigma_x - \frac{4\sigma_x^2 - \ln(\bar{\gamma})}{\sqrt{32}\sigma_x} \right) \right. \\ & \left. + \sum_{k=1}^{K} \frac{(-1)^{k+1}}{k} erfcx\left(\sqrt{2}\sigma_x + \frac{4\sigma_x^2 - \ln(\bar{\gamma})}{\sqrt{32}\sigma_x} \right) \right] \\ & - \frac{B_{fso}(4\sigma_x^2 - \ln(\bar{\gamma}))}{2\ln(2)} erfc\left(\frac{4\sigma_x^2 - \ln(\bar{\gamma})}{\sqrt{32}\sigma_x} \right) \\ & + \frac{2\sigma_x B_{fso}}{\sqrt{\pi}\ln(2)} \exp\left(- \frac{\left[4\sigma_x^2 - \ln(\bar{\gamma})\right]^2}{32\sigma_x^2} \right) \end{aligned} \qquad (14)$$

where K is a sufficiently large integer so that a value of $K \geq 8$ provides a good approximation [8]. In (14), $erfcx(x)$ is called the *scaled complementary error function* given by $erfcx(x) = e^{x^2} erfc(x) = 2e^{x^2} (\pi)^{-1/2} \int_x^\infty \exp(-r^2) dr$. Note that expression (14) is different than Eq. (4) in [13] and Eq. (10) in [8] due to a difference in the definition of lognormal PDF in (13). We can realize by Jensen's inequality that (14) is always less than the capacity of a non-fading channel with the same average power [2].

For a fading channel modeled as Gamma-Gamma distribution, the PDF in terms of SNR would be given by

$$f_I(\gamma) = \frac{\gamma^{\frac{(\alpha+\beta)}{4}-1}}{\Gamma(\alpha)\Gamma(\beta)} \left(\frac{\alpha\beta}{\sqrt{\bar{\gamma}}} \right)^{\frac{(\alpha+\beta)}{2}} K_{\alpha-\beta}\left(2\sqrt{\alpha\beta\sqrt{\gamma/\bar{\gamma}}} \right) \qquad (15)$$

for any $\gamma \geq 0$. Finally, the closed-form expression for the capacity will be expressed by [8]

$$\begin{aligned} C_{fso,GG} = & \frac{B_{fso}}{4\pi\ln(2)\Gamma(\alpha)\Gamma(\beta)} \left(\frac{\alpha\beta}{\sqrt{\bar{\gamma}}} \right)^{\frac{\alpha+\beta}{2}} \\ & \times G_{2,6}^{6,1}\left[\frac{(\alpha\beta)^2}{16\bar{\gamma}} \middle| \begin{matrix} -\frac{\alpha+\beta}{4}, -\frac{\alpha+\beta}{4}+1 \\ \frac{\alpha-\beta}{4}, \frac{\alpha-\beta}{4}+\frac{1}{2}, \frac{\beta-\alpha}{4}, \frac{\beta-\alpha}{4}+\frac{1}{2}, \frac{-\alpha-\beta}{4}, \frac{-\alpha-\beta}{4} \end{matrix} \right] \end{aligned} \qquad (16)$$

C. Capacity of Combined Link when Markov State is Available

When CSI is available, we can say that the Markov state in Fig. 1 is also available, as instantaneous SNR is also known. For independent and identically distributed (i.i.d.) fading with constant transmit power, CSI information at the transmitter has no capacity benefit. Goldsmith and Varaiya in [2] have investigated the capacity improvement for time-varying fading channels. The analysis can be applied for the considered hybrid FSO/RF design, as shown below.

We can assume X_n in (4) to be a stationary and ergodic stochastic process representing the Markov channel state. If X_n is available at both sides, the capacity of one of such time varying channel in Fig. 1 can be expressed by averaging on the capacity using stationary probabilities:

$$\bar{C} \triangleq \sum_{i=0}^{2} C_i \, p_i(s) \qquad (17)$$

where C_i denotes the capacity of a particular state S_i in (4) and $p_i(s)$ denotes the stationary probabilities discussed earlier. For a lognormal fading channel, $\{p_i(s)\}_{i=0}^2$ are represented by Eqs. (6) through (8). Considering a hybrid FSO/RF system model, it is clear that having information at state S_1 does not denote a type of connection link—either FSO or RF. Applying a simple averaging for state S_1, the overall averaged capacity of a hybrid FSO/RF link then yields to

$$\bar{C}_{fso,L} = C_{fso,L}\pi_n(0) + \frac{C_{rf} + C_{fso,L}}{2}\pi_n(1) + C_{rf}\pi_n(2) \qquad (18)$$

where $C_{fso,L}$ is represented by Eq. (14). C_{rf} is the capacity of RF link. A similar expression can be used for the capacity of a Gamma-Gamma fading channel

$$\begin{aligned} \bar{C}_{fso,GG} = & C_{fso,GG}\pi_n(0) \\ & + \frac{C_{rf} + C_{fso,GG}}{2}\pi_n(1) + C_{rf}\pi_n(2) \end{aligned} \qquad (19)$$

where in (19) $\pi_n(0)$ and $\pi_n(2)$ are defined by Eqs. (11) and (10), respectively. Also, $\pi_n(1) = 1 - \pi_n(0) - \pi_n(2)$.

D. Capacity with Power Adaptation Technique

Suppose the transmitter has access to the instantaneous CSI information of the channel and the transmit power $S(\gamma)$ can vary with the instantaneous received SNR γ with average power P_t. By Jensen's inequality representing $E[f(x)] \geq f(E[x])$, it is clearly understandable that

$$\int_{\gamma} S(\gamma) f_I(\gamma) d\gamma \leq P_t \qquad (20)$$

Adaptive transmission schemes appear in a regime where the transmit power adaptation is applied at the transmitter. In this case the Shannon-based capacity expression in (12) is no longer applicable. It has been shown [2] that the power adaptation maximizing capacity is given by

$$S(\gamma) = \begin{cases} P_t\left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right), & \gamma \geq \gamma_0 \\ 0, & 0 \leq \gamma < \gamma_0 \end{cases} \qquad (21)$$

where γ_0 is called the *cutoff* value of SNR, as the solution of

$$\int_{\gamma_0}^{\infty} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right) f_I(\gamma) d\gamma = 1 \qquad (22)$$

Eq. (22) can be numerically solved for a fading channel modeled as lognormal or Gamma-Gamma distribution; however, we merely characterize the lognormal channel. For a lognormal case, (22) is expressed using (13) by

$$\int_{\gamma_0}^{\infty} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right) f_I(\gamma) d\gamma = \frac{1}{\sqrt{32\pi}\sigma_x} \times \int_{ln\left[\frac{\gamma_0}{\bar{\gamma}}\right]}^{\infty} \left(\frac{1}{\gamma_0} - \frac{e^{-u}}{\bar{\gamma}}\right) exp\left[-\frac{(u+4\sigma_x^2)^2}{32\sigma_x^2}\right] du \qquad (23)$$

Using the help of [13, Eq. 17], we can show that

$$\int_{u_0}^{\infty} (ae^{-ky} + b)\, exp\left[-\frac{(\xi y + \mu)^2}{2\sigma^2}\right] dy = \sqrt{\frac{\pi}{2}} \frac{a\sigma}{\xi} e^{-ku_0} exp\left[-\frac{(\mu + \xi u_0)^2}{2\sigma^2}\right] erfcx\left[\frac{\sigma k}{\sqrt{2}\xi} + \frac{\mu + \xi u_0}{\sqrt{2}\sigma}\right] + \sqrt{\frac{\pi}{2}} \frac{b\sigma}{\xi} erfc\left[\frac{\mu + \xi u_0}{\sqrt{2}\sigma}\right] \qquad (24)$$

then the final expression for (22) will be equal to

$$\gamma_0 = \frac{-1}{\sqrt{8}} exp\left[-\frac{\left(4\sigma_x^2 + \ln(\gamma_0/\bar{\gamma})\right)^2}{32\sigma_x^2}\right] \times erfcx\left[\frac{3\sigma_x}{\sqrt{2}} + \frac{\ln(\gamma_0/\bar{\gamma})}{\sqrt{32}\sigma_x}\right] + \frac{1}{\sqrt{8}} erfc\left[\frac{\sigma_x}{\sqrt{2}} + \frac{\ln(\gamma_0/\bar{\gamma})}{\sqrt{32}\sigma_x}\right] \qquad (25)$$

Solving (25) for γ_0 can be found using a simple numerical root–finding method, e.g. bisection [14]. By knowing γ_0, the optimal capacity is then calculated by

$$\hat{C}_{fso} = \int_{\gamma_0}^{\infty} B_{fso} \log_2\left(\frac{\gamma}{\gamma_0}\right) f_I(\gamma) d\gamma \qquad (26)$$

which will be equal to

$$\hat{C}_{fso} = \frac{B_{fso}}{\sqrt{32\pi} ln(2) \bar{\gamma} \sigma_x} \int_{ln\left[\frac{\gamma_0}{\bar{\gamma}}\right]}^{\infty} (\ln(\bar{\gamma}/\gamma_0) + y) \times exp\left[-\frac{(y + 4\sigma_x^2)^2}{32\sigma_x^2}\right] dy \qquad (27)$$

Thus, we can show, using the help of [13, Eq. 13], that

$$\int_{u_0}^{\infty} (ay + b)\, exp\left[-\frac{(\xi y + \mu)^2}{2\sigma^2}\right] dy = \frac{a\sigma^2}{\xi^2} exp\left[-\frac{(\mu + \xi u_0)^2}{2\sigma^2}\right] - \sqrt{\frac{\pi}{2}} \sigma \left(a\mu + a\xi u_0 - \frac{b}{\xi}\right) erfc\left[\frac{\mu + \xi u_0}{\sqrt{2}\sigma}\right] \qquad (28)$$

The capacity will be given by

$$\hat{C}_{fso} = \sqrt{\frac{2}{\pi}} \frac{B_{fso}}{\ln(2)} \left\{ \sigma_x exp\left[-\frac{(4\sigma_x^2 + \ln(\gamma_0/\bar{\gamma}))^2}{32\sigma_x^2}\right] - (4\sigma_x^2 + ln(\gamma_0/\bar{\gamma}))\, erfc\left[\frac{\sigma_x}{\sqrt{2}} + \frac{\ln(\gamma_0/\bar{\gamma})}{\sqrt{32}\sigma_x}\right] \right\} \qquad (29)$$

Because a reconfigurable FSO network outage occurs when the SNR of FSO link drops to a threshold value γ_L, the averaged capacity of FSO channel using (29) can be expressed as

$$\bar{C}_{fso} = \hat{C}_{fso}\, p\left(h \geq \sqrt{\frac{\gamma_L}{\bar{\gamma}}}\right) \qquad (30)$$

where the probability $p\left(h \geq \sqrt{\frac{\gamma_L}{\bar{\gamma}}}\right)$ for a lognormal channel is given by (7) as

$$p\left(h \geq \sqrt{\frac{\gamma_L}{\bar{\gamma}}}\right) = \frac{1}{2} - \frac{1}{2} erf\left[\frac{1}{\sqrt{8}\sigma_x}\left(\ln\left(\sqrt{\frac{\gamma_L}{\bar{\gamma}}}\right) + 2\sigma_x^2\right)\right] \qquad (31)$$

If we assume that the system switches to the RF backup link when $\gamma < \gamma_0$, the total capacity of the hybrid FSO/RF system will be given by

$$\bar{C}_h = \hat{C}_{fso} p(\gamma \geq \gamma_L) + C_{rf} p(\gamma < \gamma_L) = (\hat{C}_{fso} - C_{rf})\, p(\gamma \geq \gamma_L) + C_{rf} \qquad (32)$$

We then need to calculate the value of $p(\gamma \geq \gamma_L)$ to realize the system capacity by (32). Note that (7) may not work in this scenario. Remembering (1), the instantaneous received SNR will be expressed by

$$\gamma = \frac{4h^2 R^2 S^2(\gamma)}{(\sigma_1 + \sigma_0)^2} \qquad (33)$$

By substituting $S(\gamma)$ from (21) into (33), a complex equation will be derived which will be solved for γ. Let's use an alternate logic, as follows. We know that $\gamma \geq \gamma_0$. Thus, (33) will be rewritten by

$$\sqrt{\gamma}(\sigma_1 + \sigma_0) = 2hRP_t\left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right) \qquad (34)$$

Also, we know that reconfiguration occurs when $\gamma < \gamma_L$. Thus,

$$2hRP_t \geq \sqrt{\gamma}(\sigma_1 + \sigma_0) \geq \sqrt{\gamma_L}(\sigma_1 + \sigma_0) \qquad (35)$$

Since equality is possible, (33) yields a limit for channel coefficient as

$$h \geq \frac{\sqrt{\gamma_L}(\sigma_1 + \sigma_0)}{2RP_t} = \sqrt{\gamma_L/\bar{\gamma}} \qquad (36)$$

The average capacity of a hybrid FSO/RF will then be given by

$$\bar{C}_h = (\hat{C}_{fso} - C_{rf})\, p\left(h \geq \sqrt{\frac{\gamma_L}{\bar{\gamma}}}\right) + C_{rf} \qquad (37)$$

IV. Numerical Simulation

The normalized capacity of an individual FSO link as $\hat{C}_{fso}$ in Eq. (29) has been plotted vs. SNR in Fig. 2, where the threshold SNR $\gamma_L = 5$ dB. For the power adaptation case, it can be realized from (25) that γ_0 is only a function of σ_x and $\bar{\gamma}$. Thus, the value of γ_0 can be easily found when the values for σ_x and $\bar{\gamma}$ are known, practically implementable through a lookup table. As Fig. 3 shows, not surprisingly, the capacity of an FSO link is improved when the power adaptation technique is used. However, additional challenges and complexities in hardware implementation may occur. The normalized average capacity of hybrid FSO/RF links, $\bar{C}_h/B_{fso}$ from Eq. (37), is also shown in Fig. 4, when $\gamma_L = 5$ dB. To evaluate this capacity, some assumptions about the capacity of RF links are needed. We assume the RF link is always available. Since the bandwidth of RF link is lower than FSO, it can be assumed that a constant capacity of 40 percent of the FSOs at $\sigma_x = 0.1$ and SNR=10 dB. The capacity of hybrid link for SNR values close to γ_L is influenced by the RF link.

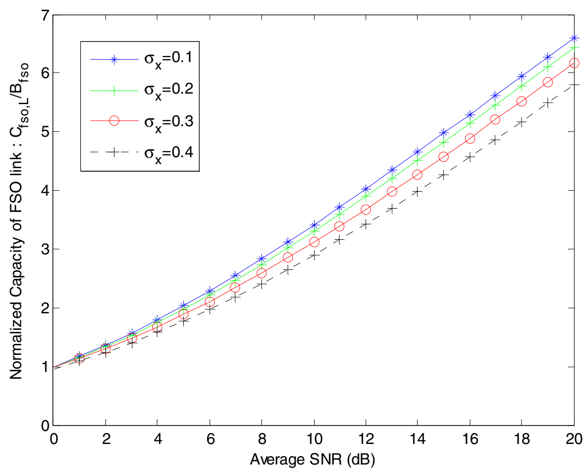

Fig. 2: The normalized capacity $C_{fso,L}/B_{fso}$ in Eq. (14) vs. averaged SNR for four different values of fading strength σ_x of a lognormal FSO channel.

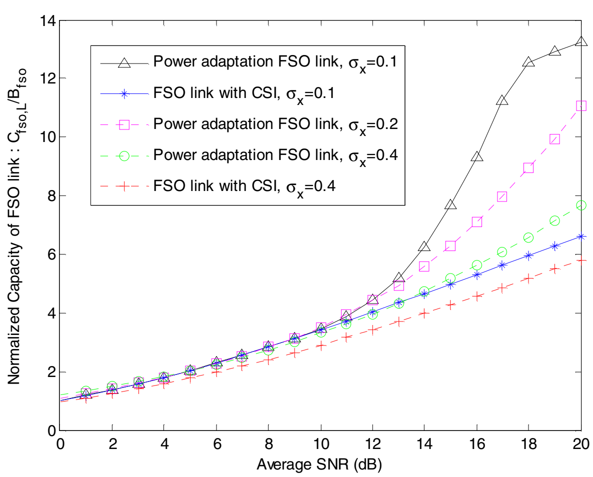

Fig. 3: A comparison between the normalized capacity $C_{fso,L}/B_{fso}$ using power adaptation technique and individual FSO capacity with various values of fading strength σ_x of a lognormal FSO channel.

V. Conclusions

Reconfigurable hybrid FSO/RF systems have been emerged to provide high reliable wireless communication in the last mile connections. But the Shannon–Hartley based capacity of FSO link is dependent on the weather conditions, mostly modeled as a turbulence-induced random fading. Authors in this paper provided new expressions for channel capacity when individual FSO communication is used in a lognormal or Gamma-Gamma fading channel. When instantaneous CSI information and thus Markov state of the channel is available, the capacity of reconfigurable hybrid FSO/RF link can be characterized. This paper, additionally, has also evaluated the effect of power adaptation technique on the capacity of combined link. A closed form expression is also derived. The improvement level is demonstrated for a lognormal optical fading channel.

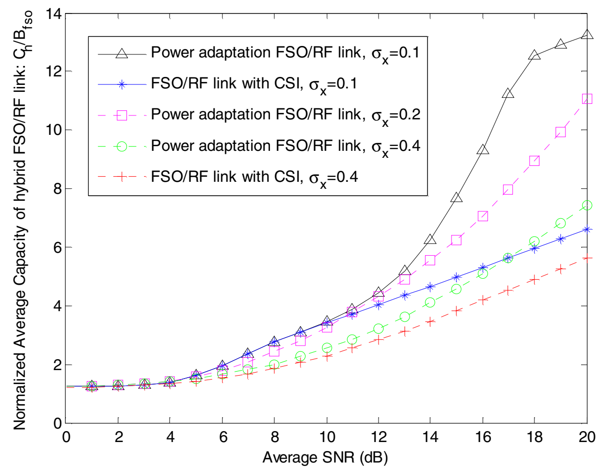

Fig. 4: Normalized average capacity of two hybrid FSO/RF links ($\bar{C}_h/B_{fso}$ in Eq. (37)) with various different values of σ_x, where $\gamma_L = 5$ dB.

References

[1] Gurumani, S., Moradi, H., Refai, H. H., LoPresti, P. G., and Atiquzzaman, M., Dynamic Path Reconfiguration among Hybrid FSO/RF Nodes, *IEEE GLOBECOM'08*, pp. 1-5, Nov. 2008.

[2] Goldsmith, A., and Varaiya, P., Capacity of fading channels with channel side information, *IEEE Transactions on Information Theory*, pp.:1986-1992, November 1997.

[3] Kashyap, A., and Shayman, M., Routing and Traffic Engineering in Hybrid RF/FSO Networks, *IEEE ICC'05*, Vol. 5, pp. 3427-3433, 2005.

[4] Jahir, Y., Atiquzzaman, M., Refai, H., and LoPresti, P., Multipath hybrid Ad hoc networks for avionics applications in disaster area, *DASC '09*, pp. 5.E.2(1-11), 2009.

[5] Sadeghi, P., Kennedy, R. A., Rapajic, P. B., and Shams, R., Finite-state Markov modeling of fading channels - a survey of principles and applications, *IEEE Signal Processing Magazine*, Volume 25, Issue 5, pp. 57-80, 2008.

[6] Moradi, H., Falahpour, M., Refai, H. H., LoPresti, P. G., and Atiquzzaman, Reconfiguration Modeling of Reconfigurable Hybrid FSO/RF Links, *IEEE ICC'10,* pp. 1-5, May 2010.

[7] —, Toward an Optimal Combined FSO/RF System via an Adaptive Bit Rate Control, *SPIE Photonics West,* Jan. 2010.

[8] Nistazakis , H. E., Karagianni, E. A., Tsigopoulos, A. D., Fafalios, M. E., and Tombras, G. S, Average Capacity of Optical Wireless Communication Systems Over Atmospheric Turbulence Channels, *Journal of Lightwave Technology*, Vol. 27, No. 8, 2009.

[9] Wang, D., and Abouzeid, A. A., Throughput Capacity of Hybrid Radio Frequency and Free Space Optical (RF/FSO) Multi-Hop Networks, *ITA'07*, pp. 3-10, 2007.

[10] Liu, B., Liu, A., and Towsley, D., On the capacity of hybrid wireless networks, *IEEE INFOCOM'03*, March 2003.

[11] Frigyes, I., and Csurgai-Horvath, L. Free-Space Optics and E-Band Radio: Complementary Techniques for Gbit/Sec Wireless, *WCNC'10 Workshops*, pp. 1-5, Sydney, Australia, 2010.

[12] Vangala, S., and Pishro-Nik, H., Optimal Hybrid RF-Wireless Optical Communication for Maximum Efficiency and Reliability, *41st CISS*, pp. 684–689, 2007.

[13] Laourine, A., Stephenne, A., and Affes, S., Capacity of lognormal fading channels, *IWCMC*, pp. 13-17, 2007.

[14] Burden, R. L., and Faires, J. D., *Numerical Analysis*, 8th ed., Thomson Learning Inc., 2008.