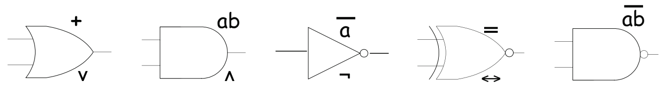
Truth Tables for Logical Operators

P	Q	P ∧ Q	P ∨ Q	P ⊻ Q	P→Q	P↔Q	¬ P
False	False	False	False	False	True	True	True
False	True	False	True	True	True	False	
True	False	False	True	True	False	False	False
True	True	True	True	False	True	True	

Combinational Gate Symbols for Logical Operators

with conventional and EE notation for operations



Inference Rules: Propositional Calculus

$$\frac{a \quad b}{a \wedge b}\{\wedge I\} \qquad \frac{a \wedge b}{a}\{\wedge E_L\} \qquad \frac{a \wedge b}{b}\{\wedge E_R\}$$

$$\frac{a}{a \vee b}\{\vee I_L\} \qquad \frac{b}{a \vee b}\{\vee I_R\} \qquad \frac{a \vee b \quad a \vdash c \quad b \vdash c}{c}\{\vee E\}$$

$$\frac{a \vdash b}{a \to b}\{\to I\} \qquad \frac{a \quad a \to b}{b}\{\to E\}$$

$$\frac{a}{a}\{ID\} \qquad \frac{\text{False}}{a}\{CTR\} \qquad \frac{\neg a \vdash \text{False}}{a}\{RAA\}$$

Some Theorems in Rule Form

$$\frac{a \wedge b}{b \wedge a}\{\wedge Comm\}$$
And Commutes

$$\frac{a \vee b}{b \vee a}\{\vee Comm\}$$
Or Commutes

$$\frac{}{a \vee (\neg a)}\{noMiddle\}$$
Law of Excluded Middle

$$\frac{a \to b \quad b \to c}{a \to c}\{\to Chain\}$$
Implication Chain Rule

$$\frac{\neg a}{a \to \text{False}}\{def'n\ \neg_F\}$$
Not Fwd

$$\frac{\neg(a \vee b)}{\neg(b \vee a)}\{\neg(\vee)Comm\}$$
Not Or Commutes

$$\frac{a \to b \quad \neg b}{\neg a}\{modTol\}$$
Modus Tollens

$$\frac{a \to \text{False}}{\neg a}\{def'n\ \neg_B\}$$
Not Bkw

$$\frac{a \to b}{(\neg b) \to (\neg a)}\{conPos_F\}$$
Contrapositive Fwd

$$\frac{a \quad \neg a}{\text{False}}\{\&\neg\}$$
NeverBoth

$$\frac{a \to b}{(\neg a) \vee b}\{\to_F\}$$
Implication Fwd

$$\frac{(\neg a) \vee b}{a \to b}\{\to_B\}$$
Implication Bkw

More Theorems in Rule Form

$$\frac{\neg(a \vee b)}{(\neg a)\wedge(\neg b)}\{DeM\vee_F\}$$

DeMorgan Or Fwd

$$\frac{(\neg a)\wedge(\neg b)}{\neg(a \vee b)}\{DeM\vee_B\}$$

DeMorgan Or Bkw

$$\frac{\neg(a \wedge b)}{(\neg a)\vee(\neg b)}\{DeM\wedge_F\}$$

DeMorgan And Fwd

$$\frac{(\neg a)\vee(\neg b)}{\neg(a \wedge b)}\{DeM\wedge_B\}$$

DeMorgan And Bkw

$$\frac{a \vee b \qquad \neg a}{b}\{disjSyll\}$$

Disjunctive Syllogism

$$\frac{\neg(\neg a)}{a}\{\neg\ \neg_F\}$$

Double Negation Fwd

$$\frac{a}{\neg(\neg a)}\{\neg\ \neg_B\}$$

Double Negation Bkw

Some Equations of Boolean Algebra

Axioms

a ∧ False = False	{∧ *null*}
a ∨ True = True	{∨ *null*}
a ∧ True = a	{∧ *identity*}
a ∨ False = a	{∨ *identity*}
a ∧ a = a	{∧ *idempotent*}
a ∨ a = a	{∨ *idempotent*}
a ∧ b = b ∧ a	{∧ *commutative*}
a ∨ b = b ∨ a	{∨ *commutative*}
(a ∧ b) ∧ c = a ∧ (b ∧ c)	{∧ *associative*}
(a ∨ b) ∨ c = a ∨ (b ∨ c)	{∨ *associative*}
a ∧ (b ∨ c) = (a ∧ b) ∨ (a ∧ c)	{∧ *distributes over* ∨}
a ∨ (b ∧ c) = (a ∨ b) ∧ (a ∨ c)	{∨ *distributes over* ∧}
¬(a ∧ b) = (¬ a) ∨ (¬ b)	{*DeMorgan's law* ∧}
¬(a ∨ b) = (¬ a) ∧ (¬ b)	{*DeMorgan's law* ∨}
¬True = False	{*negate True*}
¬False = True	{*negate False*}
(a ∧ (¬ a)) = False	{∧ *complement*}
(a ∨ (¬ a)) = True	{∨ *complement*}
¬(¬ a) = a	{*double negation*}
(a ∧ b) → c = a → (b → c)	{*Currying*}
a → b = (¬ a) ∨ b	{*implication*}
a → b = (¬ b) → (¬ a)	{*contrapositive*}

Theorems

(a ∧ b) ∨ b = b	{∨ *absorption*}
(a ∨ b) ∧ b = b	{∧ *absorption*}
(a ∨ b) → c = (a → c) ∧ (b → c)	{∨ *imp*}