Student Name: ____________________ Student ID # ____________________

UOSA Statement of Academic Integrity

On my honor I affirm that I have neither given nor received inappropriate aid in the completion of this exercise.

Signature: ____________________ Date: ____________________

Notes Regarding this Examination

Open Book(s) You may consult any printed textbooks in your immediate possession during the course of this examination.

Open Notes You may consult any printed notes in your immediate possession during the course of this examination.

No Electronic Devices Permitted You may not use any electronic devices during the course of this examination, including but not limited to calculators, computers, and cellular phones. All electronic devices in the student's possession must be turned off and placed out of sight (for example, in the student's own pocket or backpack) for the duration of the examination.

Violations Copying another's work, or possession of electronic computing or communication devices in the testing area, is cheating and grounds for penalties in accordance with school policies.

Question E2-1: Quantified Natural Deduction (10 points)

Prove the following theorem using natural deduction:

$$\forall x.(F(x) \rightarrow (\exists y.G(y))), \forall z.\neg G(z) \vdash \neg\exists x.F(x)$$

Question E2-2: Quantified Equational Reasoning (10 points)

Prove the following equation using equational reasoning:

$$\neg\exists y.(\text{even}(y) \wedge \text{odd}(y)) = \forall x.((\text{even}(x) \rightarrow \neg\text{odd}(x)) \wedge (\text{odd}(x) \rightarrow \neg\text{even}(x)))$$

Question E2-3: Induction (20 points)

Theorem {length ++ ++}:

For all sequences xs and zs, (length (xs ++ [y] ++ zs)) = (1 + length (xs ++ zs))

Prove Theorem {length ++ ++} using induction.

Hint: Use induction on n = length xs and let zs be arbitrary.

Question E2-4: Induction Redux (20 points)

Theorem {length ++ comm}:

For all sequences xs and ys, (length (xs ++ yz)) = (length (ys ++ xs))

Prove Theorem {length ++ comm} using induction.

Hint: Use induction on n = length xs and let ys be arbitrary.

For this proof, you are not permitted to use {++.additive} as a rule. However, you are permitted to use other theorems from this exam, such as {length ++ ++} and {++[]R}, even if you have not proven them.

Question E2-5: More Induction Redux (20 points)

Theorem {length ++ $*$2}:

$\forall$n.((length ([x_1, x_2, ... x_n] ++ [x_1, x_2, ... x_n])) = (2 $*$ length [x_1, x_2, ... x_n]))

Prove Theorem {length ++ $*$2} using induction.

For this proof, you are not permitted to use {++.additive} as a rule. However, you are permitted to use other theorems from this exam, such as {length ++ ++} and {++[]R}, even if you have not proven them.

Question E3-1: Sets (10 points)

Prove the following equation involving sets A, B, and C:

$$((A \cup C) - B) = ((A - B) \cup (C - B))$$

Question E3-2: Circuit Minimization using Karnaugh Maps (10 points)

Consider the following Boolean function F.

a	b	c	d	F(a,b,c,d)	minterm
0	0	0	0	x	
0	0	0	1	0	
0	0	1	0	0	
0	0	1	1	1	
0	1	0	0	0	
0	1	0	1	0	
0	1	1	0	1	
0	1	1	1	1	
1	0	0	0	0	
1	0	0	1	1	
1	0	1	0	0	
1	0	1	1	x	
1	1	0	0	0	
1	1	0	1	0	
1	1	1	0	1	
1	1	1	1	1	

A. Write the minterms of F as given by the function definition above.

B. Create a Karnaugh map for F.

C. Use the Karnaugh map to find a minimum sum-of-products representation of F, give that representation, and show how you arrived at the representation from the map.

Question E3-3: Induction (10 points)

Theorem $\{\text{len}>0\}$: For all sequences xs, length xs ≥ 0
Prove Theorem $\{\text{len}>0\}$ using induction.

Question E3-4: Induction (10 points)

Theorem {++[]R}: $\forall n.(([x_1, x_2, \ldots x_n] \mathbin{++} []) = [x_1, x_2, \ldots x_n])$
Prove Theorem {++[]R} using induction.

Question E3-5: Strong Induction (30 points)

Theorem {Prime Factorization}: $\forall n \geq 2 \in \mathbb{N}. F(n) \equiv (\text{prime}(n) \vee \exists p,q \in \mathbb{N}.(n = p * q \wedge F(p) \wedge F(q)))$
Prove Theorem {Prime Factorization} given:

$\forall n \geq 2 \in \mathbb{N}.(\text{prime}(n) \veebar \exists a,b \in \mathbb{N}.(n = a * b \wedge 1<a<n \wedge 1<b<n))$

Hint: What you are trying to prove is that every natural number greater than or equal to two is either a prime or it is a product of two other natural numbers that are each either prime or the product of two other natural numbers, etc. What you are given is that every natural number is either prime or the product of two other natural numbers that are smaller than it but still greater than or equal to two. Use this given fact to break the inductive proof into two cases: The prime case and the non-prime case.

(Additional space to complete Question E3-5.)

Question E3-6: Computation Time (20 points)

Prove by induction using our computational model that for all sequences xs, dropWhile (==0) xs terminates.

Question E3-7: Numerical Systems (30 points)

Prove using induction on n:
$\forall n \in \mathbb{N}. \forall m \in \mathbb{N}. \forall b \in \mathbb{N}_{\geq 2}.((m > n) \rightarrow ((\text{length } (\text{dgtsFromNat } b\ m)) \geq (\text{length } (\text{dgtsFromNat } b\ n))))$
Hints: With m arbitrary, prove the inequality using the antecedent of the implication as a given. You may use Theorem {len>0} from Question E3-3.
Clearly indicate in your proof every place the antecedent is assumed.

Question E3-8: Induction (10 points)

Prove using induction on n: sum[1, 2, ... n] = n ∗ (n + 1) / 2

Question E3-9: Strong Induction (40 points)

Consider the following definition:

min (m, n) =	if (m<n)	m	{min m}
	else	n	{min n}

Prove that for all sequences xs and ys, length (zipWith b xs ys)) = min (length xs, length ys).
For this proof, you may assume the following:

((length xs) $<$ (length ys)) $\leftrightarrow$ ((length (x:xs)) $<$ (length (y:ys)))

You may prove this assumption for five bonus points.

(Additional space to complete Question E3-9.)

Extra space if needed.