Inference Rules: Propositional Calculus

$$\frac{a \qquad b}{a \wedge b}\{\wedge I\} \qquad \frac{a \wedge b}{a}\{\wedge E_L\} \qquad \frac{a \wedge b}{b}\{\wedge E_R\}$$

$$\frac{a}{a \vee b}\{\vee I_L\} \qquad \frac{b}{a \vee b}\{\vee I_R\} \qquad \frac{a \vee b \qquad a \vdash c \qquad b \vdash c}{c}\{\vee E\}$$

$$\frac{a \vdash b}{a \rightarrow b}\{\rightarrow I\} \qquad \frac{a \qquad a \rightarrow b}{b}\{\rightarrow E\}$$

$$\frac{a}{a}\{ID\} \qquad \frac{\mathsf{False}}{a}\{CTR\} \qquad \frac{\neg a \vdash \mathsf{False}}{a}\{RAA\}$$

Some Theorems in Rule Form

$$\frac{a \wedge b}{b \wedge a}\ \{\wedge Comm\}$$

And Commutes

$$\frac{a \vee b}{b \vee a}\ \{\vee Comm\}$$

Or Commutes

$$\frac{}{a \vee (\neg a)}\ \{noMiddle\}$$

Law of Excluded Middle

$$\frac{a \to b \qquad b \to c}{a \to c}\ \{\to Chain\}$$

Implication Chain Rule

$$\frac{\neg(a \vee b)}{\neg(b \vee a)}\ \{\neg(\vee)Comm\}$$

Not Or Commutes

$$\frac{a \to b \qquad \neg b}{\neg a}\ \{modTol\}$$

Modus Tollens

$$\frac{a \to b}{(\neg b) \to (\neg a)}\ \{conPos_F\}$$

Contrapositive Fwd

$$\frac{}{False}\ \{\neg\&\neg\}$$

NeverBoth

$$\frac{a \to b}{(\neg a) \vee b}\ \{\to_F\}$$

Implication Fwd

$$\frac{(\neg a) \vee b}{a \to b}\ \{\to_B\}$$

Implication Bkw

More Theorems in Rule Form

$$\frac{\neg(a \lor b)}{(\neg a)\land(\neg b)}\{DeM\lor_F\}$$

DeMorgan Or Fwd

$$\frac{(\neg a)\land(\neg b)}{\neg(a \lor b)}\{DeM\lor_B\}$$

DeMorgan Or Bkw

$$\frac{\neg(a \land b)}{(\neg a)\lor(\neg b)}\{DeM\land_F\}$$

DeMorgan And Fwd

$$\frac{(\neg a)\lor(\neg b)}{\neg(a \land b)}\{DeM\land_B\}$$

DeMorgan And Bkw

$$\frac{a \lor b \qquad \neg a}{b}\{disjSyll\}$$

Disjunctive Syllogism

$$\frac{\neg(\neg a)}{a}\{\neg\ \neg_F\}$$

Double Negation Fwd

$$\frac{a}{\neg(\neg a)}\{\neg\ \neg_B\}$$

Double Negation Bkw

Some Equations of Boolean Algebra

Axioms

$a \wedge \text{False} = \text{False}$	{∧ *null*}
$a \vee \text{True} = \text{True}$	{∨ *null*}
$a \wedge \text{True} = a$	{∧ *identity*}
$a \vee \text{False} = a$	{∨ *identity*}
$a \wedge a = a$	{∧ *idempotent*}
$a \vee a = a$	{∨ *idempotent*}
$a \wedge b = b \wedge a$	{∧ *commutative*}
$a \vee b = b \vee a$	{∨ *commutative*}
$(a \wedge b) \wedge c = a \wedge (b \wedge c)$	{∧ *associative*}
$(a \vee b) \vee c = a \vee (b \vee c)$	{∨ *associative*}
$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$	{∧ *distributes over* ∨}
$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$	{∨ *distributes over* ∧}
$\neg(a \wedge b) = (\neg a) \vee (\neg b)$	{*DeMorgan's law* ∧}
$\neg(a \vee b) = (\neg a) \wedge (\neg b)$	{*DeMorgan's law* ∨}
$\neg\text{True} = \text{False}$	{*negate True*}
$\neg\text{False} = \text{True}$	{*negate False*}
$(a \wedge (\neg a)) = \text{False}$	{∧ *complement*}
$(a \vee (\neg a)) = \text{True}$	{∨ *complement*}
$\neg(\neg a) = a$	{*double negation*}
$(a \wedge b) \rightarrow c = a \rightarrow (b \rightarrow c)$	{*Currying*}
$a \rightarrow b = (\neg a) \vee b$	{*implication*}
$a \rightarrow b = (\neg b) \rightarrow (\neg a)$	{*contrapositive*}

Theorems

$(a \wedge b) \vee b = b$	{∨ *absorption*}
$(a \vee b) \wedge b = b$	{∧ *absorption*}
$(a \vee b) \rightarrow c = (a \rightarrow c) \wedge (b \rightarrow c)$	{∨ *imp*}