On the offered skills in Heroes of Might and Magic III

dimis

January 6, 2008

Abstract

In this document I will try to describe, to the best of my knowledge, the work and the results that are available online on problems related to the offered skills while leveling up a hero in *HOMM-III*. A new result is presented in section 4.4 as well as two unified tables with probabilities for hero classes; namely tables 3 and 4. Those that are interested in the problem might want to have a look on [9] which deals with a similar problem but is not within the scope of the document.

1 Instead of introduction

I guess everyone who reads this document is familiar with the game *Heroes of Might and Magic III* (*HOMM-III* for short). If not, details are given in section A in the appendix. I will summarize, to the best of my knowledge, the work and the results that are available online about matters regarding skills, and skills offered during level-ups. For me everything started with the thread *Heroes' Stats and Skills Chances* [11]. In that thread some weight tables were presented and are these weights that influence the skills that appear during level-ups. The way the weights are used for determining the offered skills is presented in section 2. Since we are dealing with a weight table, it is inevitable that we have some sort of randomness. However, practice indicates that heroes might get *exactly* the same skills as in a previous game. Therefore, the weight table alone is simply not enough to justify skill-advancing given that the implied state-space is huge [3]. That *extra* information we need is a *model* on how things work. More on that can be found in section 2. However, before I proceed I need a definition which is central throughout the document.

Definition 1.1. *Skill-advancing* or (abusively) *level-up* is the process in which a hero gains one or more new skills.

1.1 Why is it important to know about skill-advancing?

Perhaps the most significant part of the game is actually played on the way you level-up your (main) hero. The time that you accept or refute certain skills has an impact until the end of the game. And basically, all end-fights (as well as all previous fights against the map) are judged by the skills the main heroes have at that moment. Hence, the development of the heroes is the most crucial part of the game. This gives us a motive to spend our time with problems related to skill-advancing.

1.2 How do we know about the weight table and the model?

The weight table can be found in file `HCTRAITS.TXT` which is located in a bigger *container* file `H3BITMAP.LOD`. It is unclear who discovered it first, or if it was "*discovered*" due to a leak by someone working at 3DO during the early days of the game. Regarding the model, the situation is more complicated. What we know about the model comes from careful observations while playing, and there are ways in which we validate or refute our understanding of the model. More on that will appear in section 2.

1.3 Questions and problems

There are two main goals. The first one has to do with our understanding on the offered skills while leveling up a hero; i.e. what other parameters apart from the weight table influence the offers the user faces. The other one has to do with evaluating the strategy the user follows while selecting specific skills and refuting others, i.e. computing the probability specific skills (or all of them) have under a specific strategy. The work that has been done in both directions will be presented in the subsequent sections.

2 The model and how it works

A recurring theme in Computer Science is the use of weight tables in decision problems. The idea is that weights on options generate non-uniform probability distributions and usually more interesting problems. Let's see an example:

Example 1 (Biased Coin)**.** We have a biased coin and a weight function $w : \{\mathsf{H}, \mathsf{T}\} \to \mathbb{N}$ which expresses this bias. Let $w(\mathsf{H}) = 3$ and $w(\mathsf{T}) = 1$. The respective weight table is:

option	weight
Head	3
Tails	1

The important thing is the way you can compute the probability of each outcome. The most common way of doing so, is by dividing the weight of the option of interest by the sum of weights of all *available* options. Hence, the probability that Head occurs is: $\Pr[\mathsf{H}] = \frac{w(\mathsf{H})}{\sum_{s \in \{\mathsf{H},\mathsf{T}\}} w(s)} = \frac{w(\mathsf{H})}{w(\mathsf{H})+w(\mathsf{T})} = \frac{3}{3+1} = \frac{3}{4}$. Similarly, $\Pr[\mathsf{T}] = \frac{1}{4}$. □

This is the way the weights are used in *HOMM-III*. Another way of using weights is by *softmax* selection methods which usually imply Boltzmann-Gibbs distributions [12]. However, this idea was refuted in [7].

So far, most of the work that has been done, focuses on *mighty* heroes since these heroes are preferred in human games. The only attempt that I know so far which focuses on *magic* heroes as well, is [10]. However, the project is incomplete and very limited results are available.

2.1 The basic mechanism on offered skills

Each time a hero acquires a new level he is presented with two skills. The first (left) option will be a skill that he has already obtained but is not at Expert level so far. The second (right) option will be a skill that he has *not* already obtained.

Example 2. Assume that you have a Barbarian who already possesses Advanced Offense and Basic Tactics and is about to acquire a new level. During this level-up, the left offer will be either Expert Offense or Advanced Tactics and nothing else. The right offer, can be any other skill *apart from* Offense and Tactics which have already been obtained, as well as Necromancy and Water Magic, both of which are skills that Barbarians can't learn. The reason that Barbarians can't get either Necromancy or Water Magic is justified in section 2.2. Concluding, in order to give an example, one possibility in this level-up could be the pair Expert Offense and Basic Fire Magic. □

Remark 2.1. In order to fill in the gaps, let's see what other possibilities are there. The hero might have already upgraded the existing skills at Expert level and has at least one slot available for a new skill. Then, both skills that are going to be offered are (different in between) skills that have not been already acquired by the hero. Another option is that the hero might have already obtained 8 different skills and at least two are not at Expert level. In this case, the hero will be presented with 2 different skills for upgrade; i.e. no new skill. Finally, if the hero already possesses 8 skills and 7 of them are at Expert level, then he will be presented with a single option; i.e. upgrade the only skill that is not at Expert level so far. Of course, once all 8 skills reach Expert level, the hero is no longer offered any skill on each subsequent level-up.

This basic mechanism gives a rough overview of the procedure of *skill-advancing*. What is missing is a way of computing the probabilities for various skill-offers. This is the topic of the following paragraphs.

2.2 The weight tables

The weight tables which were presented in [11] and are used by default in the game are tables 1 and 2. Table 1 describes the weight distribution on the various skills for *mighty* heroes, while table 2 which deals with *magic* heroes is presented for completeness, since, as it has already been stated, no worth-mentioned results have been obtained while focusing on *magic* heroes.

Remark 2.2. The sum of all weights for each hero class is equal to 112. It is unknown why this number was picked by the designers of the game. In [13] this is falsely interpreted as:

> ... you will be offered skill choices (not only new ones, but to advance and expertise in a skill) 112 times before you cannot learn skills any further!

The statement is false based on the facts presented in section 2.1; no need for weight tables!

Remark 2.3. In [7] it is stated that by setting the weight values for specific skills greater than or equal to 128, then these weights are treated like zeros (0).

	Hero Class								
Skill	Alchemist	Barbarian	Beastmaster	Death Knight	Demoniac	Knight	Overlord	Planeswalker	Ranger
Air Magic	4	3	1	2	2	3	1	2	1
Archery	5	7	7	5	6	5	6	8	8
Armorer	8	6	10	5	7	5	6	5	8
Artillery	4	8	8	5	5	5	8	8	6
Ballistics	6	8	7	7	7	8	7	8	4
Diplomacy	3	1	1	2	4	4	3	2	4
Eagle Eye	3	2	1	4	3	2	2	2	2
Earth Magic	3	3	3	4	3	2	3	3	3
Estates	4	2	1	0	3	6	4	3	2
Fire Magic	1	2	0	1	4	1	2	3	0
First Aid	2	1	6	0	2	2	1	1	3
Intelligence	4	1	1	5	2	1	1	1	2
Leadership	3	5	5	0	3	10	8	3	6
Learning	10	4	4	4	4	4	4	8	4
Logistics	6	7	8	5	10	5	8	8	5
Luck	2	3	2	1	2	3	1	2	6
Mysticism	4	3	2	4	2	2	3	3	3
Navigation	3	2	8	8	4	8	4	5	3
Necromancy	0	0	0	10	0	0	0	0	0
Offense	6	10	5	7	8	7	8	9	5
Pathfinding	4	8	8	4	4	4	5	6	7
Resistance	5	6	5	5	6	5	6	2	9
Scholar	3	1	1	2	2	1	1	1	1
Scouting	4	8	7	4	5	4	5	6	7
Sorcery	3	1	1	4	3	1	2	1	2
Tactics	4	8	6	5	6	7	10	8	5
Water Magic	2	0	2	3	1	4	0	2	3
Wisdom	3	6	4	3	6	3	2	2	2

Table 1: The weight table for *mighty* heroes.

Skill	Hero Class								
	Battle Mage	Cleric	Druid	Elementalist	Heretic	Necromancer	Warlock	Witch	Wizard
Air Magic	3	4	2	6	3	3	2	2	6
Archery	4	3	5	2	4	2	2	3	2
Armorer	4	3	3	1	4	2	1	4	1
Artillery	4	2	1	1	4	3	1	1	1
Ballistics	6	4	4	4	6	5	6	8	4
Diplomacy	3	7	4	4	3	4	4	2	4
Eagle Eye	5	6	7	8	4	7	8	10	8
Earth Magic	3	3	4	6	4	8	5	3	3
Estates	1	3	3	3	2	3	5	1	5
Fire Magic	3	2	1	6	5	2	3	3	2
First Aid	4	10	7	4	5	0	6	8	7
Intelligence	5	6	7	8	6	6	8	7	10
Leadership	4	2	2	3	2	0	3	1	4
Learning	4	4	4	4	4	4	4	4	4
Logistics	9	4	5	2	3	4	2	3	2
Luck	2	5	9	2	2	1	2	4	4
Mysticism	4	4	6	8	10	6	8	8	8
Navigation	0	5	2	4	2	5	4	6	1
Necromancy	0	0	0	0	0	10	0	0	0
Offense	8	4	1	1	4	3	1	2	1
Pathfinding	4	2	5	2	4	6	2	2	2
Resistance	4	2	1	0	3	1	0	0	0
Scholar	4	6	8	8	5	6	8	7	9
Scouting	4	3	2	2	3	2	2	2	2
Sorcery	6	5	6	8	6	6	10	8	8
Tactics	5	2	1	1	4	2	1	1	1
Water Magic	3	4	4	6	2	3	2	3	3
Wisdom	6	7	8	8	8	8	10	8	10

Table 2: The weight table for *magic* heroes.

2.2.1 Some extra details

There are two more results from [7] which have not been presented so far.

Remark 2.4. If the hero has a skill at basic or advanced level weighing 0, it is treated as 1 when an upgrade offer is to be selected. Otherwise it is (of course) treated as 0.

Remark 2.5. If only one skill can be upgraded and that skill has an odds value of 0 or 1, *no* random number is generated to determine this offer. The skill is offered *automatically*. But if that skill has an odds value of 2 or greater the next random number will be used even though the skill is the only possible offer.

Corollary 2.6. *Hence there is a notable difference: In the latter case a random number in the sequence is actually "skipped" compared to the first case.*

2.2.2 Basic ingredient: the *weighted die*

Let's see an example of the idea that is presented in this section. There is nothing new; everything has been presented in [3] and [11].

Example 3. Assume that you have a Barbarian who has already obtained Advanced Offense and Basic Tactics and is about to acquire a new skill. How can you compute the probability of any possible combination of skill-offers? In particular, what is the probability that this hero will be offered Expert Offense and Basic Fire Magic in this level-up?

The answer comes in two parts. First, the left offer has to be a skill that has been already attained since not all of them are at Expert level. The probability that Offense has to appear is $\frac{w(\text{Offense})}{w(\text{Offense})+w(\text{Tactics})}$. Based on table 1 we have that $\Pr[\text{Offense}] = \frac{10}{18} = \frac{5}{9}$. Second, the right offer has to be a skill *not* already obtained, since the hero has free slots for additional skills. Let S be the set of all skills not already obtained by the hero; i.e. all of the skills apart from Offense and Tactics. Then by table 1, $w(S) = \sum_{s\in S} w(s) = 94$. Therefore, the probability that Fire Magic is offered as a new skill is $\Pr[\text{Fire Magic}] = \frac{2}{94} = \frac{1}{47}$. Hence, the probability that this hero will be offered Expert Offense and Basic Fire Magic in this level-up, is $\Pr[\text{Offense}] \cdot \Pr[\text{Fire Magic}] = \frac{5}{9} \cdot \frac{1}{47} = \frac{5}{423}$. □

The above example presents what is the so called "weighted-die" idea due to the resemblance of computing the probabilities with the outcome of an oddly-weighted die (biased coin with many sides if you prefer). However, experience with the game provides indications that this is not very accurate. The main reason is that the above description actually implies a huge number of possible combinations of skill-offers. On the other hand, players typically experience *identical* (not *similar*) level-ups in their games. This is a very strong indication that something is missing.

2.3 Forming a model: Allowing *exceptions*

In fact there are some *rules* which prune a lot the number of possible combinations on skill-advancing. We call such rules as *exceptions* (to the general rule). The *weighted die* idea is the *general rule* on skill-advancing, and *exceptions* force specific skills to appear here and there on level-ups. There are two kinds of exceptions:

Magic exception : A hero can not wait for more than 4 levels so that a Magic School is offered to him; either as a brand new skill or as an upgrade of an existing one.

Wisdom exception : A hero can not wait for more than 6 levels so that Wisdom is offered to him; either as a brand new skill or as an upgrade.

Remark 2.7. In [8] a method is described that re-discovers the exceptions defined above. Moreover, it is shown that it is almost certain that these are the only *exceptions* in the model.

Corollary 2.8. *Heroes that are allowed to get only 3 out of the 4 Magic Schools are able to get all those 3 Schools by simply accepting each offer that is made on Magic Schools.*

Corollary 2.9. *Barbarians, Beastmasters, Overlords, and Rangers are able to get Expert Earth Magic* always.

Remark 2.10. Note that Magic and Wisdom exception, reduce a lot the number of possible combinations of skill sequences. Still, alone this information is not enough. I 'll come back to this problem at a later section.

Example 4 (Sample Crag-Hack skill-advancing)**.** Let's see an example from actual leveling-up.

level	left offer		right offer		primary
2	Expert	Offense	**Basic**	**Tactics**	Defense
3	**Expert**	**Offense**	Basic	Fire Magic	Attack
4	**Advanced**	**Tactics**	Basic	Pathfinding	Spell Power
5	**Expert**	**Tactics**	Basic	Pathfinding	Attack
6	**Basic**	**Wisdom**	Basic	Ballistics	Defense
7	Advanced	Wisdom	**Basic**	**Earth Magic**	Attack
8	**Advanced**	**Earth Magic**	Basic	Ballistics	Attack
9	**Expert**	**Earth Magic**	Basic	Resistance	Defense
10	**Advanced**	**Wisdom**	Basic	Archery	Attack
11	**Expert**	**Wisdom**	Basic	Scouting	Knowledge
12	**Basic**	**Logistics**	Basic	Leadership	Defense
13	**Advanced**	**Logistics**	Basic	Air Magic	Knowledge
14	**Expert**	**Logistics**	Basic	Scholar	Spell Power
15	Basic	Eagle Eye	**Basic**	**Archery**	Attack
16	**Advanced**	**Archery**	Basic	Scouting	Spell Power
17	Expert	Archery	**Basic**	**Air Magic**	Defense
18	**Advanced**	**Air Magic**	Basic	Ballistics	Knowledge
19	**Expert**	**Archery**	Basic	Eagle Eye	Defense
20	**Expert**	**Air Magic**	Basic	Learning	Attack
21	Basic	Scouting	**Basic**	**Armorer**	Spell Power
22	**Advanced Armorer**				Attack
23	**Expert Armorer**				Attack

As expected, Crag-Hack is offered a Magic School before he exceeds level 4; Fire Magic at level 3. Moreover, at level 6 he is offered Wisdom due to the WISDOM exception. Note that a Magic School offer is made at most every 4 levels; e.g. Air Magic at levels 17, 18, and 20. □

2.3.1 Handling collisions

Note that it is possible for the above exceptions to coincide at some level. In [3] (posts 7 and 8 at page 2) it has been shown (with high probability: > 0.999999) that when the above exceptions coincide at some level, WISDOM exception has higher priority over MAGIC exception; meaning that if you are supposed to be offered a Magic School and Wisdom at a specific level, you will be offered only Wisdom and on the very next level you 'll get a Magic School.

Example 5 (Advancing Crag-Hack with collision)**.** *Reload* the previous position and try again:

level	left offer		right offer		primary
2	**Expert**	**Offense**	Basic	Tactics	Defense
3	**Basic**	**Archery**	Basic	Resistance	Attack
4	Advanced	Archery	**Basic**	**Fire Magic**	Spell Power
5	**Advanced**	**Archery**	Basic	Ballistics	Attack
6	**Expert**	**Archery**	Basic	Wisdom	Defense
7	**Advanced**	**Fire Magic**	Basic	Diplomacy	Attack
8	**Expert**	**Fire Magic**	Basic	Tactics	Attack
9	**Basic**	**Armorer**	Basic	Scouting	Defense
10	Advanced	Armorer	**Basic**	**Logistics**	Attack
11	**Advanced**	**Armorer**	Basic	Artillery	Knowledge
12	**Advanced**	**Logistics**	Basic	Wisdom	Defense
13	Expert	Logistics	**Basic**	**Earth Magic**	Knowledge
14	**Advanced**	**Earth Magic**	Basic	Mysticism	Spell Power
15	**Expert**	**Logistics**	Basic	Leadership	Attack
16	**Expert**	**Earth Magic**	Basic	Artillery	Spell Power
17	**Expert**	**Armorer**	Basic	Artillery	Defense
18	Basic	Wisdom	**Basic**	**Pathfinding**	Knowledge
19	**Advanced**	**Pathfinding**	Basic	Eagle Eye	Defense
20	Expert	Pathfinding	**Basic**	**Air Magic**	Attack
21	Expert	Pathfinding	**Advanced**	**Air Magic**	Spell Power
22	Expert	Pathfinding	**Expert**	**Air Magic**	Attack
23	**Expert Pathfinding**				Attack

Note that due to the WISDOM exception, Crag-Hack is offered Wisdom at level 6, and since it was rejected, again at level 12. However, note that at level 8 he was also offered Fire Magic, hence a Magic School should appear in the following 4 level-ups; i.e. in the worst case by level 12. However, since at that level only one new skill can be offered, and since WISDOM exception has higher priority over MAGIC exception, we see that Wisdom is offered at level 12 and Earth Magic at level 13. □

Remark 2.11. Inspecting examples 4 and 5 one can see that the primary skill advancement is *identical*. This was implied but I am not sure if it was clear in [7]. *Your selections on (secondary) skills do not influence primary skill advancement once the game starts.*

3 Problems that interest players

Of course the mechanism that decides the offered skills is important for game playing. But the most interesting questions w.r.t. skill-advancing deal with computing the probabilities that specific skills have to appear for certain heroes. A technical word which is used in [3] and will be used in this document as well is the word *policy*.

Definition 3.1. *Policy* is the *strategy* followed by the user when selecting specific skills on level-ups while refuting other skills. The term comes from the area of Machine Learning; [12].

Obviously the probabilities that specific skills have to appear depend on the policy the user follows. Hence, the answers (probabilities) vary as the policies vary even on the same heroes. So far work has been made on the following (deterministic) policies:

Definition 3.2 (AR or ANSA)**.** This is the *Always Right* policy; i.e. the user always picks the right offer (new skill). Initially, this policy was named as ANSA from the initials of the words Always New Skill Advancement. Both terms are used although AR should be slightly preferred. Of course this is a silly policy when playing the game. However, it has some useful properties that allow testing.

Definition 3.3 (AL)**.** This is the Always Left policy. The user constantly picks the left offer; i.e. upgrades already possessed skills to Expert level and on the following level picks the *left* offer that is made. This policy is much closer to real games than the previous one. However, there is still one silly step. In particular, when the user is presented with two new skills (Basic level) he blindly selects the left offer. Still, this policy has some useful features as well for testing the model.

There are two more interesting policies where actual work and results are provided.

Definition 3.4 (ALTP)**.** This policy is a variation from the one above. The initials come from the words Always Left Then Preference. In other words, this time, the silly step described above is omitted. Now the user declares a strict preference of skills and the skill selection is based on that preference.

Definition 3.5 (SPOU)**.** The term comes from the initials of the words Seek Preference Otherwise Upgrade. In a manner similar to ALTP the user declares a linear ordering on his preference on skills. Then, as soon as one of the top 8 skills appears as an offer, that skill is selected by the user, otherwise an already obtained skill is upgraded.

Other interesting policies are under consideration. Perhaps the most significant one is an idea due to maretti which is based on groups of skills [3]. However, so far, no policy apart from the four described above is currently implemented or concrete results are available.

4 Implementation and Results.

The main problem among most policies is the state-space that is induced by them; i.e. in most of the cases the model implies a huge amount of different combinations that have to be evaluated. However, the AR policy implies relatively few combinations of skill offers w.r.t. the computing power of modern computers; hence brute force search can compute exactly the correct values implied by the model. For the rest of the policies, where brute force computation is impossible, results reside on Monte Carlo algorithms at the moment. Let's see briefly what has been done.

4.1 AR policy

In [4] it has been shown that the problem of computing the probabilities that various skills have for any hero under this policy is tractable with brute force computations. Overall results for various hero classes are shown in table 3. Note that the numbers presented in the table are the averages among all heroes in each class. For comparison purposes, heroes that start with a certain skill, while others do not, are excluded from computing those averages. Detailed results for each hero are also available in [4] which is the home of this solver. In that same post examples are given so that someone can replicate the results in his/her own computer.

Example 6 (Averages on table 3)**.** Consider the entry on Archery on Barbarians. Since Jabarkas starts with Archery he is excluded from the average computations. Hence, 31.89 which is the number presented in table 3, is the average probability (%) for all other Barbarians. However, since all Barbarians start with Offense, it is reasonable to take into account all of them. For this reason that entry on Barbarians is 100%. □

Initially the problem was solved with the use of the `GMP`[1] library which allowed the use of fractions and hence infinite precision in computations. However, since the number of all combinations is relatively small, it can be shown that double precision arithmetic suffices for exact computations up to at least 4 decimal digits. As a result, all arithmetic operations can be done with fast registers and hence this is the program that is used nowadays in order to compute results for specific heroes. Moreover, this observation made it easy to transfer the program under Windows without the need of external libraries. Hence, if you want the program (source code and executable) make sure you download it from the link found in the paragraph `Double Precision Arithmetic and Windows 32-bit Portability` in [4]. Finally, similar results can be obtained with the program found in [5].

4.1.1 More delicate questions

Under the paragraph `Extending Computations` in [4] you can find the program `ansaExtended`. This program allows the users to ask questions about the probabilities specific combinations of skills have under the AR policy. Explanations about the questions the user is allowed to ask, as well as the syntax, and sample executions are given in that same paragraph.

[1] GNU Multi-precision arithmetic library. Home: `http://gmplib.org/`

Skill	Hero Class								
	Alchemist	Barbarian	Beastmaster	Death Knight	Demoniac	Knight	Overlord	Planeswalker	Ranger
Air Magic	46.91	43.24	19.19	25.21	25.39	36.41	19.18	25.09	16.91
Archery	22.29	31.89	31.26	24.03	28.56	23.59	27.09	35.45	34.54
Armorer	33.53	27.85	100.00	24.12	31.81	23.49	27.09	23.66	34.72
Artillery	18.18	35.60	34.91	24.12	24.23	23.49	34.66	35.64	27.08
Ballistics	26.21	35.60	31.20	32.19	32.20	35.10	30.94	35.45	18.82
Diplomacy	13.90	5.14	5.03	10.24	19.68	19.11	14.39	10.06	18.99
Eagle Eye	13.90	10.08	5.03	19.64	15.08	9.95	9.78	10.06	9.79
Earth Magic	36.15	43.24	54.40	47.55	36.97	24.95	54.38	36.58	47.98
Estates	18.18	10.08	5.03	0.00	15.08	27.56	18.81	15.19	9.79
Fire Magic	12.70	29.66	0.00	12.96	47.81	12.82	37.30	36.58	0.00
First Aid	9.44	5.14	27.31	0.00	10.26	9.95	4.98	5.13	14.40
Intelligence	18.27	5.14	5.03	24.03	10.26	5.07	4.98	5.13	9.79
Leadership	13.90	23.70	23.35	0.00	15.08	100.00	35.55	14.80	27.69
Learning	40.18	19.36	18.97	19.74	19.68	19.11	18.81	35.57	18.82
Logistics	26.21	31.79	34.89	24.03	43.13	23.41	34.58	35.53	23.17
Luck	9.44	14.82	9.86	5.22	10.26	14.63	4.98	10.06	27.17
Mysticism	18.61	14.82	9.86	19.64	10.26	9.95	14.39	14.80	14.40
Navigation	13.90	10.08	34.89	35.97	19.68	35.10	18.81	23.66	14.40
Necromancy	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
Offense	26.23	100.00	23.35	32.24	35.94	31.43	33.50	38.41	23.17
Pathfinding	18.18	35.60	34.91	19.64	19.68	19.11	23.04	27.78	30.99
Resistance	22.37	27.97	23.35	24.12	28.52	23.41	27.19	10.06	37.53
Scholar	13.56	5.14	5.03	10.24	10.36	5.07	5.04	5.13	4.99
Scouting	18.25	35.60	31.26	19.64	24.11	19.11	23.17	27.78	30.93
Sorcery	13.97	5.14	5.03	19.64	15.08	5.07	9.78	5.13	9.79
Tactics	18.25	35.60	27.31	24.12	28.52	31.43	41.24	35.98	23.05
Water Magic	24.75	0.00	37.32	36.74	13.07	47.19	0.00	25.09	47.98
Wisdom	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

Table 3: Average probabilities (%) for *mighty* heroes' classes under AR (ANSA) policy.

4.2 AL policy

This is the first policy that has been encountered in [5]. Similarly to the AR policy, table 4 presents the probabilities on average for each hero class when someone follows the AL policy. Again, in cases where one or more heroes start with a specific skill, while others in the same class do not, are excluded from computing those averages; see example 6. Results are based on a Monte Carlo method implemented in [5] and were computed with $10,000,000$ runs (episodes). Note that now that it takes longer for the 8 skills to be formed, the impact of the exceptions is more evident. Probabilities for available Magic Schools in each hero class increase, while the probabilities for all other skills decrease. Detailed results for each hero in each class can be found in [6] which is based on [5].

4.3 ALTP and SPOU policies

These are two other policies that are implemented in [5]. Note, that under these policies the user must define a strict preference on skills. Therefore, for each possibility on skill preferences the results for each hero change. Hence, it is no longer possible to create tables like 3 and 4. However, there are detailed instructions and examples in [5], so that someone can compute the probabilities that interest him.

4.4 On the validity of the results

So far there have been two (indirect) attempts on validating the theoretical results implied by the model. These two are [2] and [14]. Unfortunately, [14] seems to be incomplete. However, [2] boosted our level of confidence on validating the results. In fact, the statistical correlation between the theoretical values computed by the model and the values obtained through experimentation is 0.995. Finally, a very good source of testing level 1 to level 2 skill offers is that of Binabik in [1].

5 But is the model correct?

This is a question that is yet to be answered although some steps have been made towards this direction. The main problem is that the company who created the game has been dissolved. This makes it almost impossible for someone to have access to the original source code of the game. One way of attacking the problem would be directly with the use of decompilers and careful study of the assembly code. The other way, is trying to figure out a model for the behavior of the game and verifying (with high probability) the model with as limited experimentation as possible. This is the approach that has been followed so far.

5.1 Interesting results.

Everyone with some experience on the game could observe peculiar behavior in many cases. Random events occurred repeatedly while reloading saved positions. Several guesses have been proposed so far (page 5 in [3]) so that they can justify the strange behavior that is encountered. No definite conclusions have been drawn.

Skill	Hero Class								
	Alchemist	Barbarian	Beastmaster	Death Knight	Demoniac	Knight	Overlord	Planeswalker	Ranger
Air Magic	63.28	64.87	43.58	42.10	43.59	54.21	43.92	39.41	40.43
Archery	19.13	26.98	25.78	20.78	24.58	20.27	22.30	30.70	29.04
Armorer	29.01	23.45	100.00	20.86	27.43	20.17	22.30	20.33	29.24
Artillery	15.55	30.18	28.91	20.86	20.78	20.17	28.76	30.88	22.59
Ballistics	22.54	38.91	25.71	27.97	27.79	30.36	25.58	30.70	15.59
Diplomacy	11.86	4.26	4.04	8.78	16.82	16.36	11.70	8.58	15.75
Eagle Eye	11.86	8.37	4.04	16.93	12.85	8.47	7.92	8.58	8.05
Earth Magic	54.83	64.87	76.40	62.95	56.02	41.57	76.56	51.98	72.98
Estates	15.55	8.37	4.04	0.00	12.85	23.72	15.35	13.01	8.05
Fire Magic	23.62	52.73	0.00	23.66	64.27	23.23	66.65	51.98	0.00
First Aid	8.04	4.26	22.41	0.00	8.73	8.47	4.02	4.36	11.88
Intelligence	15.63	4.26	4.04	20.78	8.73	4.30	4.02	4.36	8.05
Leadership	11.86	19.90	19.10	0.00	12.85	100.00	29.62	12.64	23.20
Learning	34.90	16.19	15.45	17.02	16.82	16.36	15.35	30.82	15.59
Logistics	22.54	26.87	28.88	20.78	37.51	20.09	28.69	30.77	19.27
Luck	8.04	12.35	7.96	4.46	8.73	12.49	4.02	8.58	31.27
Mysticism	15.93	12.35	7.96	16.93	8.73	8.47	11.70	12.64	11.88
Navigation	11.86	8.37	28.88	31.31	16.82	30.35	15.35	20.33	11.88
Necromancy	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00
Offense	22.55	100.00	19.09	28.00	31.06	27.09	27.59	33.27	19.27
Pathfinding	15.55	30.18	28.91	16.93	16.82	16.36	18.87	23.91	25.94
Resistance	19.20	23.58	19.10	20.86	24.53	20.09	22.41	8.58	31.53
Scholar	11.56	4.26	4.04	8.78	8.84	4.30	4.08	4.36	4.09
Scouting	15.61	30.18	25.78	16.93	20.65	16.36	19.00	23.91	25.88
Sorcery	11.93	4.26	4.04	16.93	12.85	4.30	7.92	4.36	8.05
Tactics	15.61	30.18	22.41	20.86	24.53	27.09	34.50	31.22	19.14
Water Magic	42.14	0.00	66.42	54.60	25.24	62.63	0.00	39.41	72.98
Wisdom	92.30	96.25	96.34	92.01	93.91	94.84	95.02	96.29	95.05

Table 4: Average probabilities (%) for *mighty* heroes' classes under AL policy.

Another interesting idea, was that of *predefined skill sequences* by Xarfax. In fact, this idea triggered the experimentation made in [2]. Although the initial idea was (partially) refuted, we discovered that indeed a very limited number of skill sequences can be generated by the game. At that experiment, 200 episodes were generated following the AR policy, and *only* 128 ended up giving *different* sequences of skills, while a loose lower bound by the model implies well more than 1 million. However, the values of the empirical probabilities of gaining specific skills as they were formed through testing were very close to what the model predicted. As it has been already stated, their statistical correlation was 0.995. Another result, was that the ratio of previously unobserved skill sequences over the total number of sequences generated in the last x episodes converged exactly to 0.5 for $x = 20, 40$, and 80. But this ratio is close to the empirical probability of generating a skill sequence that has not been discovered so far; which implies a total of $128/0.5 = 256 = 2^8$ different skill sequences; i.e. all of them can be addressed utilizing *a single byte*.

5.2 On the validity of the model.

Under this perspective, the approval / refutation of the model (with high probability) seems perfectly doable. Approximate methods reside on the *Birthday Problem*, and variations of the *Balls in Bins Problem*. Another sort of certificate is obtained (and quantified exactly) through experiments. The outcomes between different episodes are independent; therefore by observing the discrepancy of the empirical probability on the occurrence of a specific skill from the theoretical prediction, we have a measure of confidence on validating the model. Note that we don't have just a single skill; rather than 28, which means a vector of such measures in $[0, 1]^{28} \subset \mathbb{R}^{28}$ and this makes things more complicated.

6 Future work

A more extensive experiment on AR policy thereby producing accurate values for our confidence on the model as well as make it easier to use approximate methods for verification; in other words [14] should reach an end soon. If this goes well, implement at least two more strategies on Monte Carlo methods since they have extremely practical importance: the idea on groups of skills and the idea on always picking each new Magic School offered. In addition, extensions of the implemented methods should be considered since there is a problem of who *is reaching faster* a specific skill. This is of extreme practical importance since typically heroes that break into the treasure area on random maps are between levels 11 - 14. Finally, we can switch to a learning problem: that of letting an agent find an optimal policy while trying to maximize a specific function; at the moment we are only able to *evaluate* certain policies. The implementation of neural networks on (some of) the above problems also comes into consideration.

References

[1] Binabik. $1,300$ tests from level 1 to level 2. *Library of Enlightenment*, December 2005. Post 13 at http://heroescommunity.com/viewthread.php3?TID=16647&pagenumber=1.

[2] dimis. Crag-Hack on ANSA. *Library of Enlightenment*, July 2006. Post 12 at http://heroescommunity.com/viewthread.php3?TID=17812&pagenumber=4.

[3] dimis. On the internals of offered skills when leveling-up a hero. *Library of Enlightenment*, April 2006. http://heroescommunity.com/viewthread.php3?TID=17812.

[4] dimis. The 'Always New Skill Advancement' (ANSA) problem. *Library of Enlightenment*, April 2006. Post 3 at http://heroescommunity.com/viewthread.php3?TID=17812&pagenumber=1.

[5] dimis. internals_mc: Evaluation of user's policy with Monte Carlo methods. *Library of Enlightenment*, July 2007. Post 10 at http://heroescommunity.com/viewthread.php3?TID=17812&pagenumber=8.

[6] dimis. Monte Carlo: Always Left. *Library of Enlightenment*, July 2007. Last post at http://heroescommunity.com/viewthread.php3?TID=17812&pagenumber=7.

[7] Ecoris. digging deeper - the very nature of the algorithm. *Library of Enlightenment*, August 2006. Post 2 at http://heroescommunity.com/viewthread.php3?TID=17812&pagenumber=7.

[8] Ecoris. Searching for skill groups v. 2. *Library of Enlightenment*, August 2006. Post 3 at http://heroescommunity.com/viewthread.php3?TID=17812&pagenumber=6.

[9] Ecoris. Spell probabilities by town type. *Library of Enlightenment*, May 2006. http://heroescommunity.com/viewthread.php3?TID=17964.

[10] LegendMaker. Reliable Results / Legendary Selection Method. *Library of Enlightenment*, February 2006. Post 2 at http://heroescommunity.com/viewthread.php3?TID=16647&pagenumber=2.

[11] Russ. Heroes' Stats and Skills Chances. *Library of Enlightenment*, December 2005. http://heroescommunity.com/viewthread.php3?TID=16647.

[12] R.S. Sutton and A.G. Barto. *Reinforcement Learning: An Introduction*. MIT Press, Cambridge, MA, 1998. http://www.cs.ualberta.ca/~sutton/book/ebook/the-book.html.

[13] The Nether Gods team. Secondary Skill Advancement - Heroes of Might and Magic III / Heroes 3. http://www.heroesofmightandmagic.com/heroes3/secondaryskilladv.shtml.

[14] Xarfax111. SKILLING TREE: Hack the Hack. *Library of Enlightenment*, July 2006. http://heroescommunity.com/viewthread.php3?TID=18922.

A A general framework.

There is a group of resources $\mathcal{R}$; $|\mathcal{R}| = \mathsf{N}$. There is a basket $\mathcal{B}$ with $\mathsf{M} < \mathsf{N}$ slots; each slot s_i is composed by k pockets which is constant for all slots. There is also a weight function $w_{\mathcal{B}} : \mathcal{R} \to \mathbb{N}$, which depends on the basket $\mathcal{B}$. We place resources into pockets, subject to the restriction that all k pockets in the same slot have the same resource. Moreover, different slots contain different resources. Initially l pockets are non-empty (either in the same or in different slots). Now we have the following game. The game is played with rounds. It lasts $(\mathsf{M} \cdot k - l)$ rounds and at each step we assign a resource to one of the pockets. A sequence of $(\mathsf{M} \cdot k - l)$ rounds forms an *episode*. At each step (round) there are two options (resources) and the player picks one of them to be added into the basket $\mathcal{B}$. The way the two options are presented to the player is determined by the weight function $w_{\mathcal{B}}$ and some underlying model. For the moment let's forget about the model. The two options, say a and b are presented as follows: a (left option) is a resource that can be found in some slot of the basket, but *not all* k pockets of that slot are full so far, while b (right option) is a resource that can *not* be found in any slot (pocket) of the basket so far. In the case where all k pockets are full for the resources occupying all $j < \mathsf{M}$ slots, then both a and b are resources that have not been selected so far. In the case where all M slots have filled at least 1 pocket, then both a and b are selected among the resources already in the basket for which there is an available pocket.

A.1 The main problem.

The user has a strategy (preference) when selecting resources. The computational goal is to determine the probability that each resource has to be in the basket at the end of the game according to user's strategy for various strategies.

A.2 How does the model affect the options.

There are some groups of resources, possibly all with different cardinality. Each group g has a period p_g. Now, if $p_g - 1$ steps of the game have been played and no resource $r \in g$ has been offered as an option, then on the next step of the game, we have an *exception* and one of the resources in g will be offered. The probability for each resource $r \in g$ to appear is again dependent on $w_{\mathcal{B}}$. If two (or more) exceptions coincide, then, there is a hierarchy function h which determines which exception will be enforced. All other exceptions are stored as exceptions for the next step.

A.3 The real deal.

Motivation comes from a computer game played widely on the internet. The resources are skills that can be obtained by a hero controlled by the user. The basket slots are the available slots for different skills and the pockets reflect the level of expertise of each skill. In this case it holds $|\mathcal{R}| = \mathsf{N} = 28, \mathsf{M} = 8, k = 3$, and $l = 2$. Moreover there are 3 known groups of skills, one of them (WISDOM) containing 1 resource (wisdom) with period $p_1 = 6$, the other (MAGIC) with 4 resources (air, earth, fire, water) with period $p_2 = 4$, and the third (REST) containing 23 resources with $p_3 = \infty$. The sets are disjoint. $w_{\mathcal{B}}(r) \in \{0, 1, \ldots, 10\}$, for any r and $\mathcal{B}$.