It is the large variation in constant $k$ for curve $b$ which prevents the cancellation of all the ripples.

If we designate $k_{\text{max}}$ and $k_{\text{min}}$ as the maximum and minimum values obtained for constant $k$, we can define the constant $k$ spread

$$
\xi = \frac{k_{\text{max}}}{k_{\text{min}}}
$$

which may be used as an approximate measure of the quality of the linear phase approximation. For the example at hand, curve $a$ with $\xi = 1.44$ is a considerably better approximation than curve $b$ with $\xi = 5.26$. Instead of using four relations (8) to (11) to define the zeros, we can use only one, and allow $d$ to be a positive or negative real number. Now if we prescribe different values for $d$ and calculate the resulting $\xi$, we obtain the interesting curves in Fig. 3. It seems that the best value for displacement $d$ is somewhere between 0.2 and 0.3 (or $-0.2$ and $-0.3$). The constant $k$ spread is quite low (1.3 to 1.4) in that range, so the phase angle curve could be easily maintained within the limits of $\pm 2$ to $\pm 3$ percent.

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Pseudo-Series Combination of n-Port Networks

Abstract—The "pseudo-series" combination of two n-port networks is defined. A necessary and sufficient condition is given for the combined n-port network to have an open-circuit impedance matrix equal to the sum of the corresponding matrices of the component networks.

A problem that is of interest in the synthesis of n-port networks without transformers may be stated as follows. Given an open-circuit impedance matrix $Z$, what are the conditions under which $Z$ may be expressed as the sum of $Z_1, Z_2, \ldots, Z_n$ such that each of the component matrices is conveniently realized by an n-port network, and such that a suitable combination of the component networks realizes the given $Z$-matrix? In this letter we define the pseudo-series combination of n-port networks\cite{J. Valand, "On the linear phase approximation," Proc. IEEE (Letters), vol. 55, pp. 1627-1628, September 1967.} and give a necessary and sufficient condition for the $Z$-matrix of the combined network to be equal to the sum of the $Z$-matrices of the component networks.

We consider two connected networks $N_a$ and $N_b$ having only RLC ele-
ments and identical edge and port configurations. Let \( Z_s \) and \( Z_a \) be the open-circuit impedance matrices of \( N_s \) and \( N_a \) respectively. From \( N_s \) and \( N_a \) we form a third \( n \)-port network \( N \) also having the same edge and port configurations and orientations, but having the impedance of each edge as the sum of the impedances of the corresponding edges of \( N_s \) and \( N_a \). Then \( N \) is said to be the pseudo-series combination of \( N_s \) and \( N_a \). If the open-circuit impedance matrix \( Z \) of \( N \) is equal to \( Z_s + Z_a \), then we qualify \( N \) as the proper pseudo-series combination of \( N_s \) and \( N_a \).

Let \( Z_{sa} \) and \( Z_{as} \) be the diagonal edge impedance matrices of the networks \( N_s \) and \( N_a \). Let \( B = [B_s/B_a] \) be the common fundamental circuit matrix of \( N_s \) and \( N_a \) with respect to a tree which is so chosen that all the ports are included in a cotree, and let the rows of the submatrix \( B \) correspond to the port chords and those of \( B_s \) to the nonport chords. Then we have the following as the loop-impedance matrices of \( N_s \) and \( N_a \):

\[
Z_s = \begin{bmatrix} B_s & Z_{se} \end{bmatrix} \begin{bmatrix} B_s^t & B_s \end{bmatrix} = \begin{bmatrix} Z_{11s} & Z_{12s} \\ Z_{12s}^t & Z_{22s} \end{bmatrix}
\]

(1)

\[
Z_a = \begin{bmatrix} B_a & Z_{ae} \end{bmatrix} \begin{bmatrix} B_a^t & B_a \end{bmatrix} = \begin{bmatrix} Z_{11a} & Z_{12a} \\ Z_{12a}^t & Z_{22a} \end{bmatrix}
\]

(2)

Assuming that the \( Z_{22} \) matrices are nonsingular, the modified circuit matrices \( B_s \) and \( B_a \) for the networks \( N_s \) and \( N_a \) as defined by Cederbaum \[1\] are given by

\[
B_s = B_s - Z_{12s} Z_{22s}^{-1} Z_{21s}
\]

(3)

\[
B_a = B_a - Z_{12a} Z_{22a}^{-1} Z_{21a}
\]

(4)

It can readily be shown that the pseudo-series combination of \( N_s \) and \( N_a \) is proper if their modified circuit matrices \( B_s \) and \( B_a \) are equal. \[1\] In what follows we show that the equality of the modified circuit matrices \( B_s \) and \( B_a \) is also a necessary condition in the general case for the proper pseudo-series combination of \( N_s \) and \( N_a \).

Let the pseudo-series combination of \( N_s \) and \( N_a \) be proper, yielding the combined network \( N \). We then have the following relations, where the vectors \( I_p \) and \( V_p \) refer to the port currents and voltages and the vector \( I_a \) refers to the currents in the nonport chords.

Network \( N_s \):

\[
V_p = Z_{11s} I_p + Z_{12s} I_a
\]

(5)

\[
0 = Z_{21s} I_p + Z_{22s} I_a
\]

(6)

Network \( N_a \):

\[
V_p = Z_{11a} I_p + Z_{12a} I_a
\]

(7)

\[
0 = Z_{21a} I_p + Z_{22a} I_a
\]

(8)

Network \( N \):

\[
V_p = (Z_{11s} + Z_{11a}) I_p + (Z_{12s} + Z_{12a}) I_a
\]

(9)

\[
0 = (Z_{21s} + Z_{21a}) I_p + (Z_{22s} + Z_{22a}) I_a
\]

(10)

In the foregoing, the matrices \( Z_{11s} \), \( Z_{11a} \), \( Z_{22s} \), and \( Z_{22a} \) are symmetrical, and the matrices \( Z_{12s} \) and \( Z_{12a} \) are the transposes of the matrices \( Z_{21s} \) and \( Z_{21a} \) respectively.

Since \( N \) is the proper pseudo-series combination of \( N_s \) and \( N_a \), we have \( Z_s = Z_a + Z_p \). For any arbitrary port current vector \( I_p \) we then have

\[
V_p = V_{pe} = V_{pa}
\]

(11)

leading to the following relation:

\[
Z_{12s} I_a + Z_{12a} I_a = (Z_{12s} + Z_{12a}) I_a
\]

(12)

Also, from (6), (8), and (10) we have

\[
Z_{22s} I_a + Z_{22a} I_a = (Z_{22s} + Z_{22a}) I_a
\]

(13)

Premultiplying the terms in (12) by \( I_p^t \) (the transpose of \( I_p \)) and using (6) and (8), we obtain

\[
I_a (Z_{12s} - I_a) + I_a (Z_{12a} - I_a) = 0
\]

(14)

Premultiplying the terms in (13) by \( I_p^t \), we obtain

\[
I_a (Z_{22s} - I_a) + I_a (Z_{22a} - I_a) = 0
\]

(15)

From (14) and (15) we get

\[
I_a = I_a = I_a = 0
\]

(16)

If the two matrices \( Z_{22s} \) and \( Z_{22a} \) are positive definite for real positive values of the complex frequency variable \( s \), the only way in which (16) is satisfied is when both terms on the left-hand side are zero, i.e.,

\[
I_a = I_a = I_a = 0
\]

(17)

This leads to

\[
Z_{22s} Z_{21s} I_p = Z_{22a} Z_{21a} I_p = (Z_{22s} + Z_{22a})^{-1}(Z_{21s} + Z_{21a}) I_p
\]

(18)

Since this is to be valid for all \( I_p \), it follows that

\[
Z_{12s} Z_{12a} = (Z_{12s} + Z_{12a}) (Z_{22s} + Z_{22a})^{-1}
\]

(19)

Equation (19) is true not only for real positive values of \( s \), but for all values of \( s \) by virtue of analytical continuation property. From (3), (4), and (19), it follows that the modified circuit matrices \( B_s \) and \( B_a \) of \( N_s \) and \( N_a \) are the same. It is also readily verified that \( N \) also has the same modified circuit matrix.

The foregoing discussion leads to the following theorem.

**Theorem**

The pseudo-series combination of two \( n \)-port networks \( N_s \) and \( N_a \) having nonsingular \( Z_{12} \) matrices that are positive definite for real positive values of the complex frequency variable \( s \) is proper if and only if the modified circuit matrices of \( N_s \) and \( N_a \) are equal.

It is well known that the principal minors of the loop-impedance matrices of RLC networks containing only positive resistances, inductances, and capacitances are positive definite or positive semidefinite for real positive values of \( s \). For the \( Z \)-matrix to exist, however, the \( Z_{22} \) matrices should be nonsingular and hence positive definite for real positive values of \( s \). Hence the criterion contained in the theorem is generally applicable to such networks.

The extension of the foregoing result to more than two \( n \)-port networks is obvious. A straightforward application of this criterion to test for the proper pseudo-series combination of \( p \) networks requires the inversion of \( p \) \( Z_{12} \) matrices. It can be shown, however, that the inversion of one such matrix will do for this purpose. A convenient test procedure incorporating this criterion is given elsewhere. \[3\]

Lempel and Cederbaum \[4\] have recently given a similar necessary and sufficient condition in terms of the modified cut-set matrix for the proper parallel interconnection of \( n \)-port networks without internal vertices. It can be shown that if one considers the pseudo-parallel combination instead of the regular parallel interconnection (the internal vertices are also interconnected in the pseudo-parallel combination), then the same criterion is also valid for \( n \)-port networks with internal vertices.

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