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Availability Modeling of FSO/RF Mesh Networks through Turbulence-induced Fading Channels

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***Abstract*— Even if a line-of-sight condition of Free Space Optics (FSO) is satisfied, atmospheric-induced fading, scattering, and attenuation may severely deteriorate the availability of the communication link. This argument is true except in reconfigurable FSO ad hoc networks, a path reconfiguration scheme replaces a severed FSO link with an operational one. Reconfigurability, as a property of our hybrid FSO/RF (Radio Frequency) work in progress, provides connection reliability and network throughput. The traffic can be directed to a different FSO link or even an RF link as backup. Hence, node failure and outage probabilities due to link failure will be reduced, thus resulting in a higher availability of nodes. Mathematical investigation and statistical consideration of availability and capacity of FSO/RF ad hoc mesh network is the focus of this paper. We assume normalized scintillation fading channels in our analysis. We apply different availability cases of channel state information to derive closed-form expressions for system capacity.**

***Keywords*— Node availability, link capacity, network reconfiguration, Markov model, FSO/RF ad hoc network, and fading.**

I. INTRODUCTION

FSO communication is a possible solution for IP traffic connectivity, but atmosphere turbulence, e.g. scattering, absorption and fading, highly affects the availability performance of the network. The optical links may be totally obscured by external objects, misalignment and severe weather conditions. Reinstating a failed connection, thus improving system reliability, is the objective of path reconfiguration in a configurable FSO network [1]. This is accomplished by rerouting traffic through an alternate optical link, mostly via relay nodes. In a recently proposed hybrid system known as FSO/RF [2, 3], the optical link switches to a reliable RF links as a backup.

A new probability called *reconfiguration probability* has been defined in [4] by authors which denotes the probability that a link is obscured in a *fully optical* reconfigurable ad hoc network. It is assumed that reconfiguration happens only during the time the FSO signal to noise ratio (SNR) drops below an already set value as a target threshold. Nonrandom properties of weather coming from visibility conditions will not be included in analysis, while it is investigated for a hybrid FSO/RF system by Kim and Korevaar [2]. The idea of reconfiguration has been previously examined by authors when implemented a 4-node prototype to evaluate by experiment a proposed reconfiguration algorithm [1]. The work is now in progress toward a reconfigurable hybrid FSO/RF network, supporting an adaptive rate communication [3]. We are implementing transceiver modules to build such hybrid connectivity for routing IP traffic in an ad hoc wireless network. Mathematical results in this paper help us in understanding the reliability and availability of the network.

When a link is reconfigured in a reconfigurable ad hoc network, the new path may use relay nodes to establish the new connection. As shown in Fig. 1, a new indirect FSO connection is established between nodes 1 and 3 when the direct optical connection is obscured. The new path is considered to be a fully optical connection.

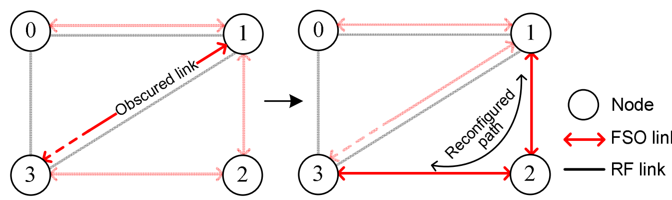

Fig.1: Reconfiguration in a simple hybrid FSO/RF computer mesh network.

As the reminder of this paper, Section II gives the system and channel model. Sections III and IV provide analytical discussions of reconfiguration probabilities for lognormal and Gamma-Gamma fading models, respectively. Section V presents an introduction of the reconfiguration analysis of FSO mesh networks. Section VI reviews a stochastic analysis toward capacity of hybrid FSO/RF links, and Section VII provides a conclusion to the paper.

II. SYSTEM AND RECONFIGURATION MODELS

We assume FSO links using intensity modulation and direct detection (IM/DD) with on/off keying (OOK). For such FSO channel with fading channel coefficient h and additive Gaussian noise, the *instantaneous* SNR will be defined by

$$\gamma \triangleq \frac{4h^2 R^2 P_t^2}{(\sigma_1 + \sigma_0)^2} \quad (1)$$

where σ_1 and σ_0 are the standard deviation of the noise currents for bits '1' and '0', respectively, R is receiver's responsivity and P_t is the average transmitted power. However, the *averaged* received SNR can be defined by

$$\bar{\gamma} \triangleq \frac{4R^2 P_t^2}{(\sigma_1 + \sigma_0)^2} \quad (2)$$

In FSO communications, fading process has a coherence time τ_c in the order of milliseconds, which is slow compared to typical symbol rates of FSO systems. A value of $T_b/\tau_c \leq 0.001$ is applicable in FSO application simulations. Thus, the channel coefficient *h* and subsequently the SNR γ can be assumed constant for a large number of transmitted bits. Hence, link outage will occur when the received SNR falls below a given threshold, say γ_L, thus a reconfiguration is required:

$$\frac{4h^2R^2P_t^2}{(\sigma_1+\sigma_0)^2} < \gamma_L \tag{3}$$

The channel coefficient *h* as the Channel State Information (CSI) is usually estimated by channel estimation and should be available through analysis. In the channel model, each state corresponds to a specific channel quality [5]. Our system model can be defined as a three-state Markov chain at any time *n*

$$X_n = \begin{cases} S_0 & \text{no reconfiguration is required at time } n \\ S_1 & \text{hysteresis gap} \\ S_2 & \text{reconfiguration is required at time } n \end{cases} \tag{4}$$

One of the stochastic components of the Markov model is the stationary (or steady) state probability vector, represented by the probability of being in Markov state *k* at any time index *n*:

$$\pi_n(S_k) = P_r(X_n = S_k) \quad k\text{=0, 1 and 2} \tag{5}$$

The reconfiguration mechanism uses two thresholds from received signal strength of optical wireless link. One threshold level called—*reconfiguration threshold*—is selected on the basis of the minimum required SNR by the receiver. The other—*switch back threshold*—is used to recover traffic transmission on the old optical wireless link. Reconfiguration is accomplished at time t_0 in [4, Fig. 2] while switch back to the original configuration begins once the received SNR improves to γ_H at time t_1.

III. Reconfiguration Probabilities in Lognormal Fading

A lognormal distribution of the fading channel will be defined by

$$f_x(h) = \frac{1}{\sqrt{8\pi}h\sigma_x}\exp\left(-\frac{\left[\ln(h)+2\sigma_x^2\right]^2}{8\sigma_x^2}\right) \tag{6}$$

where the magnitude of fading coefficient is the ratio of the faded light intensity to the intensity without turbulence $h = I_n/I_0$. It is so called as normalized irradiance. σ_x^2 is the variance of log-amplitude fading [3]. The fading coefficient in (6) is chosen normalized to provide $E[h]$=1. Recalling a lognormal distribution for normalized irradiance *h* by (6), the probability of being a link at state 2 at time *n,* which means it is either obscured or too noisy, is expressed in terms of stationary probabilities, and can be rewritten by [4]

$$\pi_n(2) = \frac{1}{2} + \frac{1}{2}erf\left[\frac{1}{\sqrt{8}\sigma_x}\left(\ln\left(\sqrt{\frac{\gamma_L}{\bar{\gamma}}}\right)+2\sigma_x^2\right)\right] \tag{7}$$

where *erf*(.) is the *error function*. We may use 2 as S_2 for simplicity in this paper. Similarly the probability $\pi_n(0)$ as the probability of recovering from an obscured link or in terms of SNR is equal to

$$\pi_n(0) = \frac{1}{2} - \frac{1}{2}erf\left[\frac{1}{\sqrt{8}\sigma_x}\left(\ln\left(\sqrt{\frac{\gamma_H}{\bar{\gamma}}}\right)+2\sigma_x^2\right)\right] \tag{8}$$

Finally, the probability of being at state 1 can be easily calculated by

$$\pi_n(1) = 1 - \pi_n(0) - \pi_n(2) \tag{9}$$

where $\pi_n(0)$ and $\pi_n(2)$ are given by (8) and (7), respectively. The probability $\pi_n(2)$ is numerically shown in Fig. 2 versus averaged received SNR for different values of σ_x, representing fading strength for $\gamma_L = 4$ dB.

IV. Reconfiguration Probabilities in Gamma-Gamma Fading

In recent years, Gamma–Gamma fading model has become foremost for FSO links because it provides acceptable matching between the model and the evaluated data coming from turbulence induced fading channel. The fading model has a pdf expressed as the form of [6]:

$$f_I(h) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)}h^{(\alpha+\beta)/2-1}K_{\alpha-\beta}\left(2\sqrt{\alpha\beta h}\right)\ ;\ h>0 \tag{10}$$

where *h* is the channel attenuation (coefficient) due to atmospheric turbulence, $K_v(.)$ is the modified Bessel function of the second kind of order *v*, $\Gamma(.)$ is the gamma function, while parameters *α* and *β* can be directly related to atmospheric conditions. Corresponding expressions can be found in [6]. If we choose *β*=1, a less complicated pdf—*K* turbulence model—is derived [7]. Again by recalling the probability distribution for normalized irradiance *h* by (6), the probability of an obscured link or excessive noise at time *n* is expressed in terms of stationary probabilities. Using [4, Eq. (14)], the reconfiguration probability will be given by

$$\pi_n(2) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)}\int_0^{\sqrt{\frac{\gamma_L}{\bar{\gamma}}}} h^{\frac{\alpha+\beta}{2}-1}K_{\alpha-\beta}\left(2\sqrt{\alpha\beta h}\right)dh \tag{11}$$

We may use Meijer's G-function to solve the integral in (11) where $K_v(x) = \frac{1}{2}G_{0,2}^{2,0}\left[x^2/2\middle|_{v/2,\,-v/2}^{-}\right]$. Then, the stationary probability results in

$$\pi_n(2) = 1 - \frac{(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)}\int_{\sqrt{\frac{\gamma_L}{\bar{\gamma}}}}^{\infty} h^{\frac{\alpha+\beta}{2}-1}G_{0,2}^{2,0}\left[\alpha\beta h\middle|_{\frac{\alpha-\beta}{2},\,\frac{\beta-\alpha}{2}}^{-}\right]dh \tag{12}$$

Using [7, Eq.(7)], a closed-form expression is derived as

$$\pi_n(2) = 1 - \frac{\alpha\beta}{\Gamma(\alpha)\Gamma(\beta)}\left(\alpha\beta\sqrt{\frac{\gamma_L}{\bar{\gamma}}}\right)^{\frac{\alpha-\beta}{2}}G_{1,3}^{3,0}\left[\alpha\beta\sqrt{\frac{\gamma_L}{\bar{\gamma}}}\middle|_{1-\frac{\alpha+\beta}{2},\,\frac{\alpha-\beta}{2},\,\frac{\beta-\alpha}{2}}^{-\frac{\alpha+\beta}{2}}\right] \tag{13}$$

Similarly the stationary probability $\pi_n(0)$ representing the normal operation of FSO link, i.e. no need for reconfiguration, can be calculated by

$$\pi_n(0) = \frac{\alpha\beta}{\Gamma(\alpha)\Gamma(\beta)}\left(\alpha\beta\sqrt{\frac{\gamma_L}{\bar{\gamma}}}\right)^{\frac{\alpha-\beta}{2}}G_{1,3}^{3,0}\left[\alpha\beta\sqrt{\frac{\gamma_L}{\bar{\gamma}}}\middle|_{1-\frac{\alpha+\beta}{2},\,\frac{\alpha-\beta}{2},\,\frac{\beta-\alpha}{2}}^{-\frac{\alpha+\beta}{2}}\right] \tag{14}$$

Fortunately, Meijer's G-function can be calculated by a variety of mathematical software packages. The function is implemented in MATHEMATICA and MAPLE as MeijerG(.) function and in MPMATH by *Python* as meijerg(.) function. But MATLAB, which was in more interest for our computations, doesn't support this function. Thus, we use Eq. (11) to numerically compute the integral. The results are shown in Figs. 3 and 4.

V. Reconfiguration in Mesh Networks

Analysis of the reliability and availability of nodes in FSO-based ad hoc mesh networks can be accomplished in a statistical perspective. A realistic fault tolerance model can be introduced in which each node in a mesh network has an independent failure probability. Without loss of generality, the reconfiguration probability between two nodes n and m in an N-node FSO mesh network, M_N, can be noted by $P_r^{(n,m)}$. A probability matrix can be defined to specify the probabilities between nodes:

$$P_{N\times N}=\begin{bmatrix} 0 & P_r^{(1,2)} & P_r^{(1,3)} & \cdots & P_r^{(1,N-1)} & P_r^{(1,N)} \\ P_r^{(2,1)} & 0 & P_r^{(2,3)} & \cdots & P_r^{(2,N-1)} & P_r^{(2,N)} \\ P_r^{(3,1)} & P_r^{(3,2)} & 0 & \cdots & P_r^{(3,N-1)} & P_r^{(3,N)} \\ \vdots & \vdots & \vdots & 0 & \vdots & \vdots \\ P_r^{(N-1,1)} & P_r^{(N-1,2)} & P_r^{(N-1,3)} & \cdots & 0 & P_r^{(N-1,N)} \\ P_r^{(N,1)} & P_r^{(N,2)} & P_r^{(N,3)} & \cdots & P_r^{(N,N-1)} & 0 \end{bmatrix} \quad (15)$$

We assumed $P_r^{(n,m)}=0$ for any n=m. This assumption can be considered for any two nodes having no connection as well. Each connection link between any two nodes n and m in the FSO mesh network fails independently with probability $P_r^{(n,m)}$, $0\le P_r^{(n,m)}<1$. Assuming different independent fading channel irradiances for different links, $P_r^{(n,m)}$ can be expressed using (7) and (13) for lognormal and Gamma-Gamma, respectively, by

$$P_{r,L}^{(n,m)}=\frac{1}{2}+\frac{1}{2}erf\left[\frac{1}{\sqrt{8}\sigma_x^{(n,m)}}\left(\ln\left(\sqrt{\frac{\gamma_H^{(n,m)}}{\bar{\gamma}^{(n,m)}}}\right)+2(\sigma_x^{(n,m)})^2\right)\right] \quad (16)$$

$$P_{r,GG}^{(n,m)}=1-\frac{\alpha_{n,m}\beta_{n,m}}{\Gamma(\alpha_{n,m})\Gamma(\beta_{n,m})}\left(\alpha_{n,m}\beta_{n,m}\sqrt{\frac{\gamma_L^{(n,m)}}{\bar{\gamma}^{(n,m)}}}\right)^{\frac{\alpha_{n,m}-\beta_{n,m}}{2}} \times G_{1,3}^{3,0}\left[\alpha_{n,m}\beta_{n,m}\sqrt{\frac{\gamma_L^{(n,m)}}{\bar{\gamma}^{(n,m)}}}\left|\begin{matrix} -\frac{\alpha_{n,m}+\beta_{n,m}}{2} \\ 1-\frac{\alpha_{n,m}+\beta_{n,m}}{2}, \frac{\alpha_{n,m}-\beta_{n,m}}{2}, \frac{\beta_{n,m}-\alpha_{n,m}}{2}\end{matrix}\right.\right] \quad (17)$$

With the above assumptions, if K_n, $K_n\le N$-1, be the number of active FSO links connected to node n, the probability that at least one of the links connecting to this node is faded by atmosphere turbulence fading can be calculated by

$$\hat{P}_r^{(n)}=1-\prod_{i=1}^{K_n}\left(1-P_r^{(n,i)}\right) \quad (18)$$

where $P_r^{(n,i)}$ is the probability of reconfiguration for the link i-th defined by (16) or (17). Also we will derive, in terms of $P_r^{(n,i)}$, the probability that a node in a fully optical networks fails. In a reconfigurable mesh network node failure happens when all of its links are disconnected. Thus, the probability of node failure in such mesh network can be expressed by

$$\widetilde{P}_r^{(n)}=\prod_{i=1}^{K_n}P_r^{(n,i)} \quad (19)$$

while $1-\widetilde{P}_r^{(n)}$ represents the connectivity of the node

$$1-\prod_{i=1}^{K_n}P_r^{(n,i)} \quad (20)$$

Expression (20) is among the most important parameters in characterizing network connection reliability. It is mostly called node *availability*. This parameter may not be close to 1 in a fully optical FSO network. However, hybrid FSO/RF networks have been introduced to enhance this parameter even to a value of 0.99999 (5 nines) [2]. The node availability based on (20) is plotted in Fig. 5 for a Gamma-Gamma fading channel.

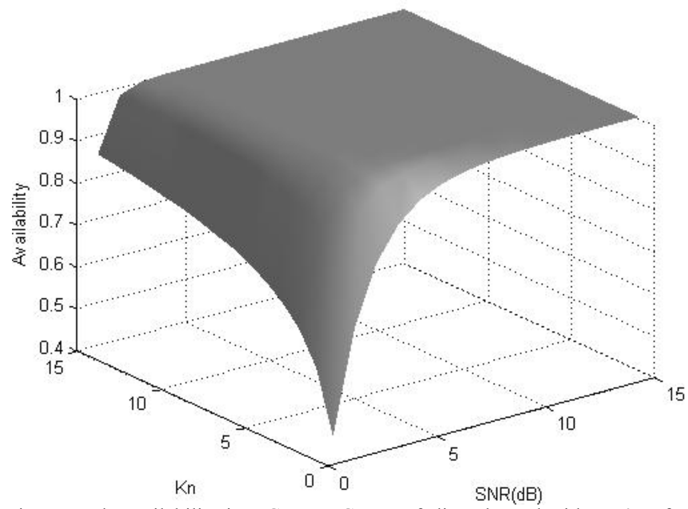

Fig. 5: Node availability in a Gamma-Gamma fading channel with α =0.5, β=5 and γ_L=4dB.

VI. Capacity Model

Performance considerations of communication systems can be also investigated through capacity analysis. The capacity of FSO based ad hoc networks comes with extensive attention because of the high capacity of FSO links but the severe sensitivity of the link availability to atmosphere condition.

A. Capacity with Perfect CSI at the Receiver Side

If CSI is available at the receiver side in a clear weather condition, the nominal averaged capacity (bits/sec) of a given FSO link is calculated by integration on the received instantaneous electrical SNR γ [8]

$$C_{fso}=\int_0^\infty B_{fso}\log_2(1+\gamma)f_x(\gamma)d\gamma \quad (21)$$

where B_{fso} is the FSO channel's bandwidth and $f_x(\gamma)$ is the fading pdf in SNR representation. For a lognormal fading model, when substituting h from (1) and (2) as $h=\sqrt{\gamma/\bar{\gamma}}$ into (6):

$$f_x(\gamma)=\frac{1}{\sqrt{32\pi}\gamma\sigma_x}\exp\left(-\frac{\left[\ln(\gamma/\bar{\gamma})+4\sigma_x^2\right]^2}{16\sigma_x^2}\right) \quad (22)$$

for any $\gamma \geq 0$. A closed form expression then can be derived for $C_{fso,L}$ using [9]:

$$C_{fso,L} = \frac{B_{fso}}{2\ln(2)} \exp\left(-\frac{\left[4\sigma_x^2 - \ln(\bar{\gamma})\right]^2}{16\sigma_x^2}\right) \times \left[\sum_{k=1}^{K} \frac{(-1)^{k+1}}{k} erfcx\left(2\sigma_x - \frac{4\sigma_x^2 - \ln(\bar{\gamma})}{4\sigma_x}\right) + \sum_{k=1}^{K} \frac{(-1)^{k+1}}{k} erfcx\left(2\sigma_x + \frac{4\sigma_x^2 - \ln(\bar{\gamma})}{4\sigma_x}\right)\right] - \frac{B_{fso}(4\sigma_x^2 - \ln(\bar{\gamma}))}{2\ln(2)} erfc\left(\frac{4\sigma_x^2 - \ln(\bar{\gamma})}{4\sigma_x}\right) + \frac{2\sigma_x B_{fso}}{\sqrt{\pi}\ln(2)} \exp\left(-\frac{\left[4\sigma_x^2 - \ln(\bar{\gamma})\right]^2}{16\sigma_x^2}\right) \quad (23)$$

where K is a sufficiently large integer ($K \geq 8$, [6]) and where $erfcx(x)$ is called the *scaled complementary error function* given by $erfcx(x) = e^{x^2} erfc(x) = 2e^{x^2} (\pi)^{-1/2} \int_x^{\infty} \exp(-r^2) dr$. Note that expression (23) is a little different than Eq. (4) in [9] and Eq. (10) in [6] due to the variation in the definition of lognormal pdf in (22). We can realize by Jensen's inequality that (23) is always less than the capacity of a non-fading channel with the same average power [8].

For a fading channel modeled as Gamma-Gamma distribution, the pdf in terms of SNR would be given from (10) as

$$f_x(\gamma) = \frac{\gamma^{\frac{(\alpha+\beta)}{4}-1}}{\Gamma(\alpha)\Gamma(\beta)} \left(\frac{\alpha\beta}{\sqrt{\bar{\gamma}}}\right)^{\frac{(\alpha+\beta)}{2}} K_{\alpha-\beta}\left(2\sqrt{\alpha\beta\sqrt{\gamma/\bar{\gamma}}}\right) \quad (24)$$

for any $\gamma \geq 0$. Finally, the closed-form expression for the capacity will be expressed by [6]

$$C_{fso,GG} = \frac{B_{fso}}{4\pi\ln(2)\Gamma(\alpha)\Gamma(\beta)} \left(\frac{\alpha\beta}{\sqrt{\bar{\gamma}}}\right)^{\frac{\alpha+\beta}{2}} \times G_{2,6}^{6,1}\left[\frac{(\alpha\beta)^2}{16\bar{\gamma}} \middle| \begin{matrix} -\frac{\alpha+\beta}{4}, -\frac{\alpha+\beta}{4}+1 \\ \frac{\alpha-\beta}{4}, \frac{\alpha-\beta}{4}+\frac{1}{2}, \frac{\beta-\alpha}{4}, \frac{\beta-\alpha}{4}+\frac{1}{2}, \frac{-\alpha-\beta}{4}, \frac{-\alpha-\beta}{4} \end{matrix}\right] \quad (25)$$

The hybrid FSO/RF ad hoc network of N nodes is crossly connected by N_{rf}-$N(N-1)/2$ FSO links and N_{rf} links, each capable of transmitting at B_{fso} bits/sec and B_{rf} bits/sec, respectively, where $B_{rf} << B_{fso}$.

B. Capacity with Markov State (but not CSI) at Both the Receiver and Transmitter Sides

For independent and identically distributed (i.i.d.) fading with constant transmit power, CSI information at the transmitter has no capacity benefit. Goldsmith and Varaiya in [8] have investigated the capacity improvement for time varying fading channels. The analysis can be applied for our hybrid FSO/RF design, as we now show.

We can readily assume that X_n in (4) be a stationary and ergodic stochastic process representing the Markov channel state. If X_n be available at both sides, the capacity of one of the such time varying channel in Fig. (1) can be expressed by averaging on the capacity using stationary probabilities:

$$\bar{C} \triangleq \sum_{i=0}^{2} C_i \, p_i(s) \quad (26)$$

where C_i denotes the capacity of a particular state S_i in (8) and $p_i(s)$ denotes the stationary probabilities discussed earlier. For a lognormal fading channel, $\{p_i(s)\}_{i=0}^{2}$ are represented by Eqs. (7) to (9). Considering a hybrid FSO/RF system model, it is clear that having the information of being at state S_1 does not denote the type of connection link, either FSO or RF. Thus applying a simple averaging for state S_1, the overall averaged capacity of a hybrid FSO/RF link then yields to

$$\bar{C}_{fso,L} = C_{fso,L}\pi_n(0) + \frac{C_{rf} + C_{fso,L}}{2}\pi_n(1) + C_{rf}\pi_n(2) \quad (27)$$

where $C_{fso,L}$ is represented by Eq. (23). Also, C_{rf} is the capacity of RF link, which is not in the scope of this paper. Same expression can be expressed for the capacity of a Gamma-Gamma fading channel

$$\bar{C}_{fso,GG} = C_{fso,GG}\pi_n(0) + \frac{C_{rf} + C_{fso,GG}}{2}\pi_n(1) + C_{rf}\pi_n(2) \quad (28)$$

where in (28) $\pi_n(0)$ and $\pi_n(2)$ are defined by Eqs. (14) and (13), respectively. Also, $\pi_n(1) = 1 - \pi_n(0) - \pi_n(2)$.

C. Capacity with Perfect CSI at Both the Receiver and Transmitter Sides

Now suppose the transmitter has also access to the instantaneous CSI information of the channel and the transmit power $S(\gamma)$ can vary with the instantaneous received SNR γ with average power P_t . By Jensen's inequality,

$$\int_{\gamma} S(\gamma) f_x(\gamma) d\gamma \leq P_t \quad (29)$$

Such adaptive transmission schemes appears in a regime where the transmit power adaptation is applied at transmitter. In this case the Shannon-based capacity expression in (21) is not applicable anymore. It has been shown [8] that the power adaptation which maximizes the capacity is given by

$$S(\gamma) = \begin{cases} P_t\left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right), & \gamma \geq \gamma_0 \\ 0, & 0 \leq \gamma < \gamma_0 \end{cases} \quad (30)$$

where γ_0 is called the *cutoff* value of SNR, as the solution of

$$\int_{\gamma_0}^{\infty} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right) f_x(\gamma) d\gamma = 1 \quad (31)$$

In this case, the optimal capacity is calculated by

$$\hat{C}_{fso} = \int_{\gamma_0}^{\infty} B_{fso} \log_2\left(\frac{\gamma}{\gamma_0}\right) f_x(\gamma) d\gamma \quad (32)$$

If we assume that the system switches to the RF backup link when $\gamma < \gamma_0$, the total capacity of the hybrid FSO/RF system will be given by

$$\bar{C} = \hat{C}_{fso} p(\gamma \geq \gamma_0) + C_{rf} p(\gamma < \gamma_0) = (\hat{C}_{fso} - C_{rf})\, p(\gamma \geq \gamma_0) + C_{rf} \quad (33)$$

where $p(.)$ is the probability function. We need to calculate the value of $p(\gamma \geq \gamma_0)$ to realize the system capacity by (33). Remembering (1), the instantaneous received SNR will be expressed by

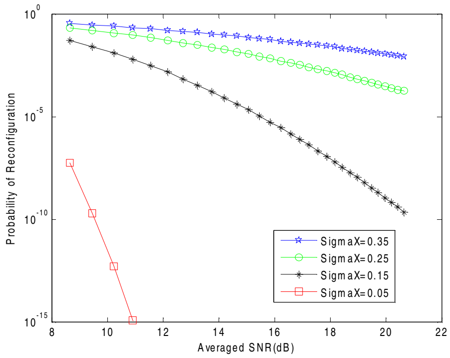

Fig. 2: Probability of reconfiguration, $\pi_n(2)$, vs. SNR for lognormal channel at different values of fading intensity. γ_L=4dB.

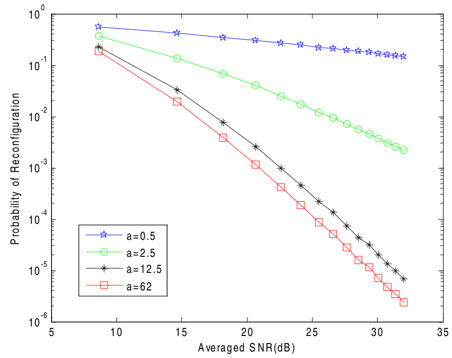

Fig. 3: Probability of reconfiguration, $\pi_n(2)$, vs. SNR for Gamma-Gamma channel with different values of α. γ_L=4dB, β=5.

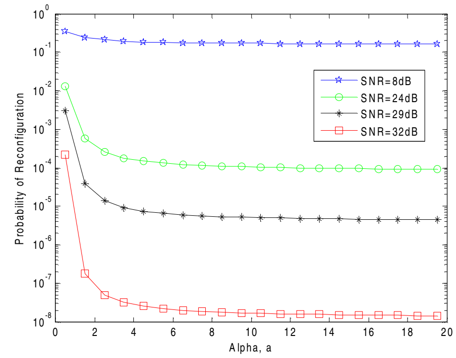

Fig. 4: Probability of reconfiguration, $\pi_n(2)$, vs. α for Gamma-Gamma channel with different values of SNR at γ_L=4dB, β=5.

$$\gamma = \frac{4h^2R^2S^2(\gamma)}{(\sigma_1+\sigma_0)^2} \tag{34}$$

By substituting $S(\gamma)$ from (30) into (34) then a complex equation will be derived which will be hard to solved for γ. Let's use a different logic as follows. We assume $\gamma \geq \gamma_0$ and then (34) will be rewritten by

$$\sqrt{\gamma}(\sigma_1+\sigma_0) = 2hRP_t\left(\frac{1}{\gamma_0}-\frac{1}{\gamma}\right) \tag{35}$$

Thus,

$$2hRP_t \geq \sqrt{\gamma}(\sigma_1+\sigma_0) \geq \sqrt{\gamma_0}(\sigma_1+\sigma_0) \tag{36}$$

And since the equality can be a possible case, (36) yields a limit for channel coefficient as

$$h \geq \frac{\sqrt{\gamma_0}(\sigma_1+\sigma_0)}{2RP_t} = \sqrt{\frac{\gamma_0}{\bar{\gamma}}} \tag{37}$$

and then the average capacity of a hybrid FSO/RF will be given by

$$\bar{C} = (\hat{C}_{fso} - C_{rf})\, p\left(h \geq \sqrt{\frac{\gamma_0}{\bar{\gamma}}}\right) + C_{rf} \tag{38}$$

where the probability $p\left(h \geq \sqrt{\frac{\gamma_0}{\bar{\gamma}}}\right)$ is given by

$$p\left(h \geq \sqrt{\frac{\gamma_0}{\bar{\gamma}}}\right) = \frac{1}{2} - \frac{1}{2}erf\left[\frac{1}{\sqrt{8}\sigma_x}\left(\ln\left(\sqrt{\frac{\gamma_0}{\bar{\gamma}}}\right)+2\sigma_x^2\right)\right] \tag{39}$$

for a lognormal fading channel, and

$$p\left(h \geq \sqrt{\frac{\gamma_0}{\bar{\gamma}}}\right) = \frac{\alpha\beta}{\Gamma(\alpha)\Gamma(\beta)}\left(\alpha\beta\sqrt{\frac{\gamma_0}{\bar{\gamma}}}\right)^{\frac{\alpha-\beta}{2}} \times G_{1,3}^{3,0}\left[\alpha\beta\sqrt{\frac{\gamma_0}{\bar{\gamma}}}\,\middle|\,{}^{-\frac{\alpha+\beta}{2}}_{1-\frac{\alpha+\beta}{2},\,\frac{\alpha-\beta}{2},\,\frac{\beta-\alpha}{2}}\right] \tag{40}$$

for a Gamma-Gamma fading channel. Further analysis on the capacity of the hybrid FSO/RF systems will be presented in a future work by authors.

VII. Conclusions

An analytical investigation for characterizing availability probabilities in FSO/RF networks was conducted. Both lognormal and Gamma-Gamma fading models have been assumed in analysis. A simple Markov model has been used for probability analysis through reconfiguration occurrence in view of received SNR. New closed-form expressions for capacity of a hybrid FSO/RF system were defined to characterize the availability links. The authors stochastically derived closed-form expressions, which describe the availability and reconfiguration process probabilities. The results demonstrate the characterization of our hybrid FSO/RF network which is under implementation.

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