Student Name: ______________________ Student ID # ______________________

UOSA Statement of Academic Integrity

On my honor I affirm that I have neither given nor received inappropriate aid in the completion of this exercise.

Signature: ______________________ Date: ______________________

Notes Regarding this Examination

Open Book(s) You may consult any printed textbooks in your immediate possession during the course of this examination.

Open Notes You may consult any printed notes in your immediate possession during the course of this examination.

No Electronic Devices Permitted You may not use any electronic devices during the course of this examination, including but not limited to calculators, computers, and cellular phones. All electronic devices in the student's possession must be turned off and placed out of sight (for example, in the student's own pocket or backpack) for the duration of the examination.

Violations Copying another's work, or possession of electronic computing or communication devices in the testing area, is cheating and grounds for penalties in accordance with school policies.

Question 1: Quantified Natural Deduction (10 points)

Prove the following theorem using natural deduction:

$$(\forall x.(F(x) \vee \neg G(x))) \vdash (\forall x.(G(x) \rightarrow F(x)))$$

Question 2: Quantified Equational Reasoning (10 points)

Prove the following equation using equational reasoning:

$$(\forall x.(F(x) \rightarrow G(x))) = (\neg \exists x.(F(x) \wedge \neg G(x)))$$

Question 3: Sets (10 points)

Prove the following equation involving sets A, B, and C:

$$((A \cup C) - B) = ((A - B) \cup (C - B))$$

Question 4: Circuit Minimization using Karnaugh Maps (10 points)

Consider the following Boolean function F.

a	b	c	d	F(a,b,c,d)	minterm
0	0	0	0	x	
0	0	0	1	0	
0	0	1	0	0	
0	0	1	1	1	
0	1	0	0	0	
0	1	0	1	0	
0	1	1	0	1	
0	1	1	1	1	
1	0	0	0	0	
1	0	0	1	1	
1	0	1	0	0	
1	0	1	1	x	
1	1	0	0	0	
1	1	0	1	0	
1	1	1	0	1	
1	1	1	1	1	

A. Write the minterms of F as given by the function definition above.

B. Create a Karnaugh map for F.

C. Use the Karnaugh map to find a minimum sum-of-products representation of F, give that representation, and show how you arrived at the representation from the map.

Question 5: Induction (20 points)

Consider the following type definitions and axioms:

m::(Num a, Natural n) $\Rightarrow$ a $\rightarrow$ n $\rightarrow$ a
m 0 a = 0 $\{m_0\}$
m (n+1) a = a + (m n a) $\{m_{n+1}\}$
plus::Num a $\Rightarrow$ a $\rightarrow$ a $\rightarrow$ a
plus a b = a + b $\{$def. plus$\}$

Prove: map (m 2) [x_1, x_2, ... x_n] = zipWith plus [x_1, x_2, ... x_n] [x_1, x_2, ... x_n]

Question 6: Induction Redux (20 points)

Consider the following type definitions and axioms:

r::(Natural n) $\Rightarrow$ a $\rightarrow$ n $\rightarrow$ [a]
r 0 x = [] $\{r_0\}$
r (n+1) x = x : (r n x) $\{r_{n+1}\}$
prod::Num a $\Rightarrow$ [a] $\rightarrow$
prod [] = 1 {prod[]}
prod (x: xs) = x * (prod xs) {prod:}
power (Num a, Natural n) $\Rightarrow$ a $\rightarrow$ n $\rightarrow$ a
power a 0 $\{pow_0\}$
power a (n+1) = a * (power a n) $\{pow_{n+1}\}$

Prove: $\forall$ n.((prod (r n x) = (power x n))

Question 7: More Induction Redux (20 points)

Consider the following type definitions and axioms:

foldl:: (a $\rightarrow$ b) $\rightarrow$ a $\rightarrow$ [b] $\rightarrow$ a
foldl ($\oplus$) z [] = z {foldl[]}
foldl ($\oplus$) z (x: xs) = foldl ($\oplus$) (z $\oplus$ x) xs {foldl:}

Prove: $\forall$ n.((foldl (+) z [x_1, x_2, ... x_n]) = (z + foldr (+) 0 [x_1, x_2, ... x_n]))