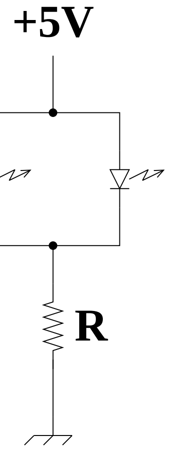
Embedded Systems (CS 503/591C)
Homework 1 Solutions

December 15, 2003

Question 1

Consider the following circuit:



Assume that R = 100 ohms and that the voltage drop across each LED is 2 Volts.

1. What is the current flow through the resistor?

   $I = \frac{\Delta V}{R} = \frac{3V}{100\Omega} = 30mA$

2. How much work does each LED perform?

   $15mA$ through each LED.

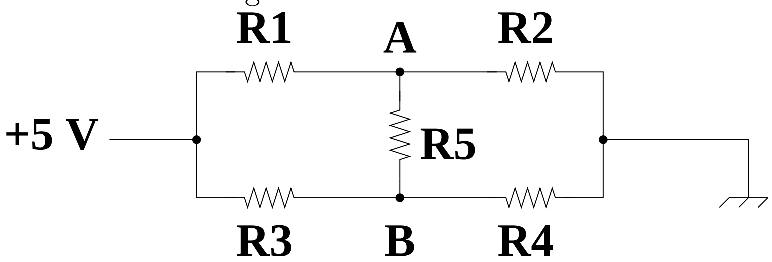
   $W = \Delta V I = 2V * 15mA = 30mW$

3. How does the work manifest itself?

   Light

Question 2

Consider the following circuit:



Assume that R1 = R5 = 10 ohms, R2 = R3 = 25 ohms, and R4 = 100 ohms.

1. What is the voltage at point A?

2. At point B?

   Note that if we know V_a and V_b, we can easily apply Ohm's law to compute current flow through the network. So - this becomes a problem of 2 unknowns and we will see that we have two equations that contain only those unknowns.

   We assume that positive current across the resistors goes to the right or down as drawn.

   Applying Kirchhoff's current law, we have:

   $I_1 = I_5 + I_2$, and

   $I_3 + I_5 = I_4$.

   Using Ohm's law, we have:

   $\frac{5-V_a}{R1} = \frac{V_a-V_b}{R_5} + \frac{V_a-0}{R_2}$, and

   $\frac{5-V_b}{R3} + \frac{V_a-V_b}{R_5} = \frac{V_b-0}{R_4}$.

   Substituting the constants:

   $\frac{5-V_a}{10} = \frac{V_a-V_b}{10} + \frac{V_a}{25}$
   $\rightarrow 25 - 5V_a = 5V_a - 5V_b + 2V_a$
   $\rightarrow 25 + 5V_b = 12V_a$, and

$\frac{5-V_b}{25} + \frac{V_a - V_b}{10} = \frac{V_b}{100}$
$\rightarrow 20 - 4V_b + 10V_a - 10V_b = V_b$
$\rightarrow 4 + 2V_a = 3V_b$

Substituting one equation into the other, we have:

$V_b = \frac{49}{13} \approx 3.77$, and

$V_a = \frac{3*49/13-4}{2} \approx 3.65$.

3. What is the effective resistance of this lattice?

   Total current flow through the input our output lines is:

   $I_2 + I_4 = \frac{V_a}{R2} + \frac{V_b}{R4} \approx 0.146 + 0.0377 = 0.1837A$

   and:

   $R_{effective} = \frac{V}{I_2+I_4} = 27.22\Omega$

4. What is the current flow through R5?

   $I = \frac{V_a - V_b}{R5} \approx -12mA$ (as measured from A to B)

5. How much work is done by R1?

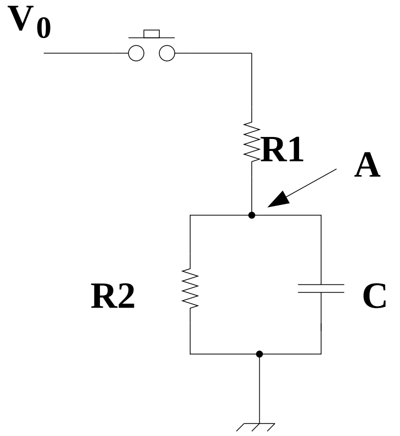
   $W = \Delta VI = \frac{\Delta V^2}{R1} = \frac{5-V_a}{10} \approx 0.183W$

6. How does this work manifest itself?

   Heat.

Question 3

Consider the following circuit:



1. With the button in the state as shown, what is the voltage at point A?

   $V_a = 0V$

2. What is the current across R1, R2 and C (assume steady state)?

   $I_1 = I_2 = I_C = 0A$

3. With the button pressed and with the circuit in steady state, what is the voltage at point A?

   In the steady state, there is no current across the capacitor. So:

   $V_a = V_0 * \frac{R2}{R1+R2}$

4. What is the current across R1, R2, and C?

   Since $I_C = 0$, $I_2 = I_1$.

   Applying Ohm's law:

   $I_2 = \frac{V_a}{R2} = \frac{V_0}{R1+R2}$.

5. Assume that the circuit is in steady state with the button in the state as shown and that at time t=0, the button is pressed. What is the temporal evolution of the voltage at point A?

   Assume that positive current is down.

   Applying Kirchhoff's current law:

$I_1 = I_2 + I_C$.

Applying Ohm's law:

$\frac{V_0 - V_a}{R1} = \frac{V_a}{R2} + C\frac{dV_a}{dt}$
$\rightarrow R2V_0 - R2V_a = R1V_a + R1R2C\frac{dV_a}{dt}$
$\rightarrow R2V_0 - (R1 + R2)V_a = R1R2C\frac{dV_a}{dt}$
$\rightarrow \frac{dt}{R1R2C} = \frac{dV_a}{R2V_0 - (R1+R2)V_a}$.

Integrating both sides:

$\frac{t}{R1R2C}|_0^t = -ln(R2V_0 - (R1 + R2)V_a)/(R1 + R2)|_0^t$
$\rightarrow \frac{-t(R1+R2)}{R1R2C} = ln\left(\frac{R2V_0 - (R1+R2)V_a(t)}{R2V_0 - (R1+R2)V_a(0)}\right)$

We know that $V_a(0) = 0$. So:

$exp\left(\frac{-t(R1+R2)}{R1R2C}\right) = \frac{R2V_0 - (R1+R2)V_a(t)}{R2V_0}$
$\rightarrow V_a(t) = V_0\frac{R2}{R1+R2}\left(1 - exp\left(\frac{-t(R1+R2)}{R1R2C}\right)\right)$.

6. What is the time constant of this evolution?

   $\frac{R1R2C}{R1+R2}$

7. What is the flow of current (as a function of time) across R1, R2, and C?

   $I_1(t) = \frac{V_0 - V_a(t)}{R1} = \frac{V_0}{R1} - V_0\frac{R2}{R1(R1+R2)}\left(1 - exp\left(\frac{-t(R1+R2)}{R1R2C}\right)\right)$

   $I_2(t) = \frac{V_a(t)}{R2} = V_0\frac{1}{R1+R2}\left(1 - exp\left(\frac{-t(R1+R2)}{R1R2C}\right)\right)$

   $I_C(t) = I_1(t) - I_2(t)$

8. Assume that the circuit is in steady state with the button pressed and that at time t=0, the button is released. What is the temporal evolution of the voltage at point A?

   This is the same case as we did in class (except for a small difference in the initial condition):

   $V_a(t) = V_0\frac{R2}{R1+R2}exp\left(\frac{-t}{R2C}\right)$

9. What is the time constant?

   $CR2$