Consider the following circuit:

\[ V_0 \]
\[ V_1 \]
\[ \text{Assume } V_0 = 5V. \]
1. (10pts) Assume $R_2 = 100\Omega$. Show $V_1$ and $I$ as a function of $1\Omega \leq R_1 \leq 299\Omega$. Show your derivation.

The circuit gives us:

\[
\begin{align*}
V_0 - V_1 &= R_1 I_{R_1} \\
V_1 - 0 &= R_2 I_{R_2} \\
I_{R_1} &= I_{R_2}
\end{align*}
\]

This gives us:

\[
\begin{align*}
\frac{V_0 - V_1}{R_1} &= \frac{V_1}{R_2} \\
V_1 &= V_0 \frac{R_2}{R_1 + R_2}
\end{align*}
\]

We will assume arbitrarily that $V_0 = 5V$.

The relationships look like this:

![Graph A](image1.png)

![Graph B](image2.png)

2. (10pts) Assume $R_2 = 300 - R_1$. Show $V_1$ and $I$ as a function of $R_1$ (same range). Show your derivation.

With this additional constraint, we have:

\[
\begin{align*}
V_1 &= V_0 \frac{300 - R_1}{300} \\
I_{R2} &= V_0 \frac{1}{300}
\end{align*}
\]

The key differences here are that $V_1$ is now a linear function of $R_1$, and $I_{R2}$ is a constant with respect to $R_1$. 

2
The relationships look like this:

This second case is a type of variable resistor that we refer to as a *linear potentiometer*. These devices have 3 pins: at \( V_0 \), \( V_1 \) and ground in our circuit. The resistance from \( V_0 \) to ground is constant (i.e., the sum of \( R_1 \) and \( R_2 \) remains constant). However, the voltage at the midpoint will vary through the range of \( V_0 \) to ground, depending on how the “wiper” is set.

**Question 2**

1. (10pts) Consider the above circuit. Assume \( R = 500 \Omega \). Show \( IR \) as a function of \( V_0 \). Show your derivation.

The circuit gives us:

\[
V_0 - 0 = RI_R
\]

3
So:

\[ I_R = \frac{V_0}{R} \]

The relationship looks like this:
2. (20pts) Consider the above circuit. Assume $V_f = 0.5V$ and $R = 500\Omega$. Show $IR$ as a function of $V0$. Show your derivation.

For any given $V_0$, there are a total of 16 different possibilities: two each for the four diodes (current flowing or not). However, we can quickly eliminate many cases. First, let’s label the diodes: top left and right (1 and 2); bottom left and right (3 and 4).

The following are always true:

\[
\begin{align*}
V_2 - V_1 &= RI_R \\
I_R &= I_{D1} + I_{D3} \\
I_{D2} + I_{D4} &= I_R
\end{align*}
\]

Consider the case in which $D_1$ and $D_3$ are both turned on. This implies that:

\[
\begin{align*}
V_1 &= 0.5V \\
V_0 &= 0V \\
I_R &= 0A
\end{align*}
\]
The latter equation implies that:

\[ I_{D2} + I_{D4} > 0A \]

This means that at least one of the two right-hand-side diodes must be turned on. If either is turned on, then:

\[ V_2 = -0.5V \]

This implies that:

\[ I_R < 0A \]

This is a contradiction. Therefore, the left-hand-side diodes cannot be turned on together. The same logic can also be applied to right-hand-side diodes.

Now consider the case where the bottom diodes are both turned on. In this case, the logic is the same as above, and we arrive at a contradiction.

Likewise, consider the case where the top diodes are both turned on. This implies that:

\[
\begin{align*}
V_1 - V_0 &= 0.5V \\
V_0 - V_2 &= 0.5V \\
I_{D1} &> 0A \\
I_{D2} &> 0A
\end{align*}
\]

However:

\[
\begin{align*}
V_1 - V_2 &= 1V \\
I_R &= -2mA
\end{align*}
\]

But - this is a contradiction to the currents flowing through the top two diodes. So - this case cannot happen.
We are left with three possible cases: no diodes are turned on, $D_1$ and $D_4$ are on together, or $D_2$ and $D_3$ are on together. The question now is: which case applies for any particular choice of $V_0$?

Assume case 2:

\[
\begin{align*}
V_1 - V_0 & = 0.5V \\
0 - V_2 & = 0.5V \\
I_{D1} & > 0A \\
I_{D4} & > 0A
\end{align*}
\]

Therefore:

\[
\begin{align*}
I_R & > 0A \\
V_2 - V_1 & > 0V
\end{align*}
\] (1)

Substituting above into this last equation, we achieve:

\[
V_0 < -1V
\] (2)

And:

\[I_R = \frac{-V_0 - 1}{R}\] (3)

Next, assume case 3:

\[
\begin{align*}
V_0 - V_2 & = 0.5V \\
V_1 - 0 & = 0.5V \\
I_{D2} & > 0A \\
I_{D3} & > 0A
\end{align*}
\]
Therefore:

\[ I_R > 0A \]
\[ V_2 - V_1 > 0V \]

Substituting above into this last equation, we achieve:

\[ V_0 > 1V \]

(5)

And:

\[ I_R = \frac{V_0 - 1}{R} \]

(6)

Case 1 applies in the remaining case:

\[ -1V \leq V_0 \leq 1V \]

(7)

And:

\[ I_R = 0A \]

(8)

The relationship looks like this:
The key is: no matter what $V_0$ is, current through the resistor is never negative. This is what we refer to as a \textit{rectifier}. 
**Question 3**

Suppose we want to produce a regular interrupt at a period of approximately 32.8 ms. Assume that we are using a 16 MHz crystal for our clock.

1. (5 pts) Which timer should we use?  
   Timer 1.

2. (5 pts) How should we configure this timer?  
   Prescaler: 8

**Question 4**

Suppose we want to produce a regular interrupt at a frequency of 1.953 KHz. Assume that we are using a 16 MHz crystal for our clock.

1. (5 pts) Which timer should we use?  
   Timer 2.

2. (5 pts) How should we configure this timer?  
   Prescaler: 32