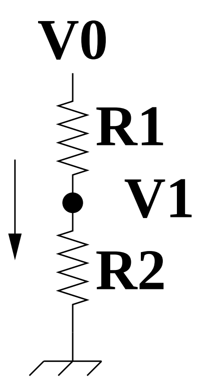
Embedded Systems (CS [45]163)
Homework 2 Solutions

March 19, 2009

Question 1

Consider the following circuit:



Assume $V0 = 5V$.

1. (10pts) Assume $R2 = 100\Omega$. Show $V1$ and I as a function of $1\Omega \leq R1 \leq 299\Omega$. Show your derivation.

   The circuit gives us:

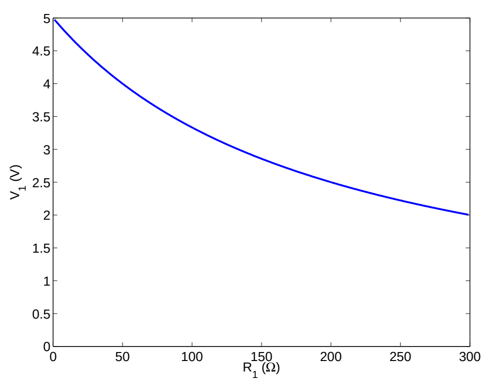
$$\begin{aligned} V_0 - V_1 &= R_1 I_{R1} \\ V_1 - 0 &= R_2 I_{R2} \\ I_{R1} &= I_{R2} \end{aligned}$$

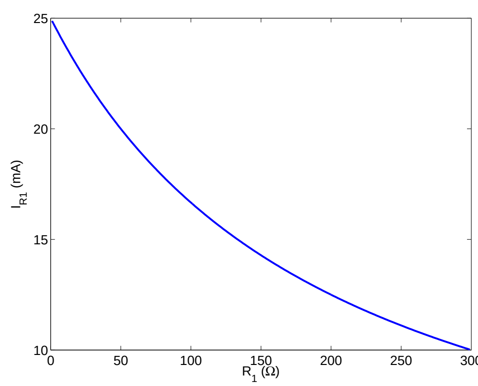
   This gives us:

$$\begin{aligned} \frac{V_0 - V_1}{R_1} &= \frac{V_1}{R_2} \\ V_1 &= V_0 \frac{R_2}{R1 + R2} \end{aligned}$$

   We will assume arbitrarily that $V_0 = 5V$.

   The relationships look like this:





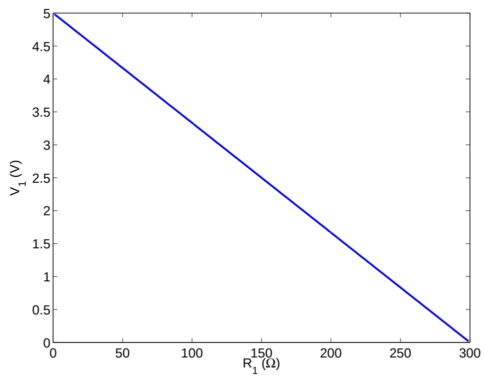
2. (10pts) Assume $R2 = 300 - R1$. Show $V1$ and I as a function of $R1$ (same range). Show your derivation.

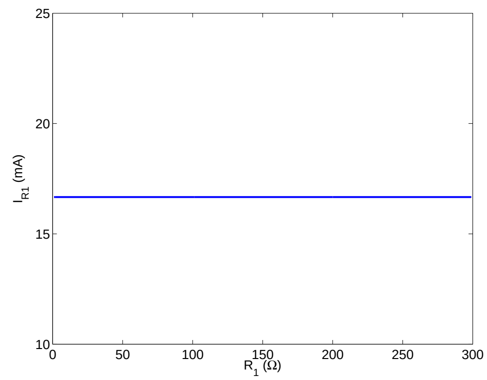
   With this additional constraint, we have:

$$\begin{aligned} V_1 &= V_0 \frac{300 - R_1}{300} \\ I_{R2} &= V_0 \frac{1}{300} \end{aligned}$$

   The key differences here are that V_1 is now a linear function of R_1, and I_{R2} is a constant with respect to R_1.

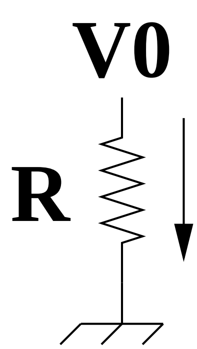
The relationships look like this:





This second case is a type of variable resistor that we refer to as a *linear potentiometer*. These devices have 3 pins: at V_0, V_1 and ground in our circuit. The resistance from V_0 to ground is constant (i.e., the sum of R_1 and R_2 remains constant). However, the voltage at the midpoint will vary through the range of V_0 to ground, depending on how the "wiper" is set.

Question 2



1. (10pts) Consider the above circuit. Assume $R = 500\Omega$. Show IR as a function of $V0$. Show your derivation.

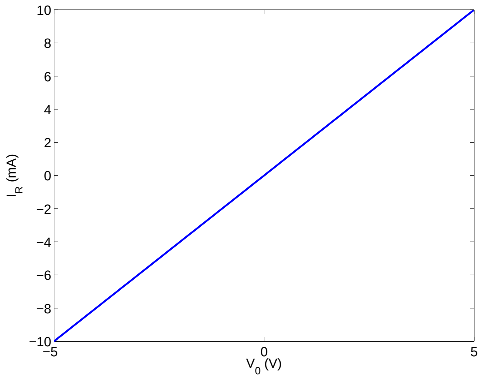
   The circuit gives us:

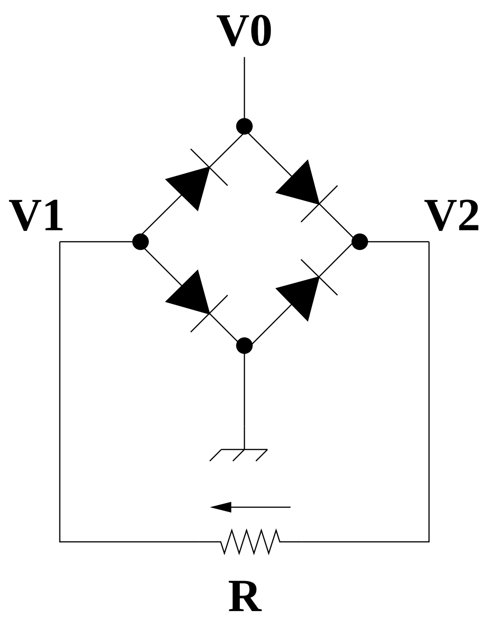
$$V_0 - 0 \quad = \quad RI_R$$

So:

$$I_R = \frac{V_0}{R}$$

The relationship looks like this:





2. (20pts) Consider the above circuit. Assume $V_f = 0.5V$ and $R = 500\Omega$. Show IR as a function of $V0$. Show your derivation.

   For any given V_0, there are a total of 16 different possibilities: two each for the four diodes (current flowing or not). However, we can quickly eliminate many cases. First, let's label the diodes: top left and right (1 and 2); bottom left and right (3 and 4).

   The following are always true:

$$\begin{aligned} V_2 - V_1 &= RI_R \\ I_R &= I_{D1} + I_{D3} \\ I_{D2} + I_{D4} &= I_R \end{aligned}$$

   Consider the case in which D_1 and D_3 are both turned on. This implies that:

$$\begin{aligned} V_1 &= 0.5V \\ V_0 &= 0V \\ I_R &> 0A \end{aligned}$$

The latter equation implies that:

$$I_{D2} + I_{D4} \quad > \quad 0A$$

This means that at least one of the two right-hand-side didoes must be turned on. If either is turned on, then:

$$V_2 \quad = \quad -0.5V$$

This implies that:

$$I_R \quad < \quad 0A$$

This is a contradiction. Therefore, the left-hand-side diodes cannot be turned on together. The same logic can also be applied to right-hand-side diodes.

Now consider the case where the bottom diodes are both turned on. In this case, the logic is the same as above, and we arrive at a contradiction.

Likewise, consider the case where the top diodes are both turned on. This implies that:

$$\begin{aligned} V_1 - V_0 &= 0.5V \\ V_0 - V_2 &= 0.5V \\ I_{D1} &> 0A \\ I_{D2} &> 0A \end{aligned}$$

However:

$$\begin{aligned} V_1 - V_2 &= 1V \\ I_R &= -2mA \end{aligned}$$

But - this is a contradiction to the currents flowing through the top two diodes. So - this case cannot happen.

We are left with three possible cases: no diodes are turned on, D_1 and D_4 are on together, or D_2 and D_3 are on together. The question now is: which case applies for any particular choice of V_0?

Assume case 2:

$$\begin{aligned} V_1 - V_0 &= 0.5V \\ 0 - V_2 &= 0.5V \\ I_{D1} &> 0A \\ I_{D4} &> 0A \end{aligned}$$

Therefore:

$$\begin{aligned} I_R &> 0A \\ V_2 - V_1 &> 0V \end{aligned} \tag{1}$$

Substituting above into this last equation, we achieve:

$$V_0 < -1V \tag{2}$$

And:

$$I_R = \frac{-V_0 - 1}{R} \tag{3}$$

Next, assume case 3:

$$\begin{aligned} V_0 - V_2 &= 0.5V \\ V_1 - 0 &= 0.5V \\ I_{D2} &> 0A \\ I_{D3} &> 0A \end{aligned}$$

Therefore:

$$\begin{aligned} I_R &> 0A \\ V_2 - V_1 &> 0V \end{aligned} \tag{4}$$

Substituting above into this last equation, we achieve:

$$V_0 > 1V \tag{5}$$

And:

$$I_R = \frac{V_0 - 1}{R} \tag{6}$$

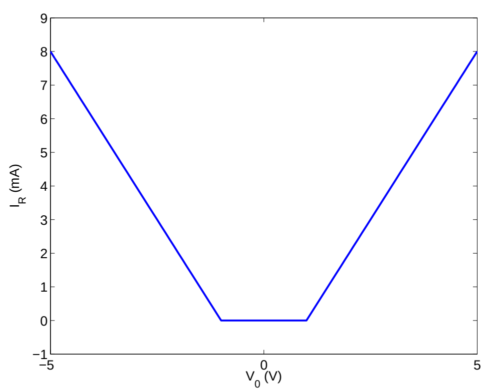
Case 1 applies in the remaining case:

$$-1V \leq V_0 \leq 1V \tag{7}$$

And:

$$I_R = 0A \tag{8}$$

The relationship looks like this:



The key is: no matter what V_0 is, current through the resistor is never negative. This is what we refer to as a *rectifier*.

Question 3

Suppose we want to produce a regular interrupt at a period of approximately 32.8 ms. Assume that we are using a 16 MHz crystal for our clock.

1. (5 pts) Which timer should we use?

   Timer 1.

2. (5 pts) How should we configure this timer?

   Prescaler: 8

Question 4

Suppose we want to produce a regular interrupt at a frequency of 1.953 KHz. Assume that we are using a 16 MHz crystal for our clock.

1. (5 pts) Which timer should we use?

   Timer 2.

2. (5 pts) How should we configure this timer?

   Prescaler: 32