Memory

• With combinatorial logic, we could only implement “stateless” functions

• By introducing flip-flops, we could remember something about the history of the inputs
Memory

• With combinatorial logic, we could only implement “stateless” functions

• By introducing sequential logic (with flip-flops), we could remember something about the history of the inputs

How do we formalize this idea of “history”?
Formalizing Memory

Combinatorial Logic  Boolean Algebra
Formalizing Memory

Combinatorial Logic    Boolean Algebra

Sequential Logic
Formalizing Memory

Combinatorial Logic

Boolean Algebra

Sequential Logic

Finite State Machines
Formalizing Memory

Combinatorial Logic       Boolean Algebra

Sequential Logic           Finite State Machines

This will allow us to express controllers that take history into account ....
Finite State Machines (FSMs)

Pure FSM form is composed of:

- A set of states
- A set of possible inputs (or events)
- A set of possible outputs
- A transition function:
  – Given the current state and an input: defines the output and the next state
Finite State Machines (FSMs)

States:

• Represent all possible “situations” that must be distinguished
• At any given time, the system is in exactly one of the states
• There is a finite number of these states
Finite State Machines (FSMs)

An example: our synchronous counter

- States: ?
Finite State Machines (FSMs)

An example: our synchronous counter

- States: the different combinations of the digits: 000, 001, 010, … 111

- Inputs: ?
Finite State Machines (FSMs)

An example: our synchronous counter

• Inputs:
  – Really only one: the event associated with the clock transitioning from high to low
  – We will call this “C”

• Outputs: ?
Finite State Machines (FSMs)

An example: our synchronous counter

- Outputs: same as the set of states

- Transition function: ?
Finite State Machines (FSMs)

An example: our synchronous counter
• Transition function:
  – On the clock event, transition to the next state in the sequence
Administrivia

• Expect to have 3 helis up today
  – Motors have been failing. We are taking steps to deal with this
  – But:
    • Minimize flight times
    • Don’t stall the motors
    • Don’t spend much time tuning PD parameters

• Getting caught up on grading this week

• Project 3 due Wednesday
Last Time

- Interrupt Service Routines (ISRs)
  - Responding quickly to events
  - Timer overflow ISRs: regular events

- Finite State Machines (FSM)
  - Intuitively expressing sequential behavior
  - Translate easily into code
FSM Example: Synchronous Counter

A Graphical Representation:

A set of states
FSM Example: Synchronous Counter

A transition

C/001

000

001

010

011

100

101

110

111
FSM Example: Synchronous Counter

A transition

The event
FSM Example: Synchronous Counter

A transition

The output
FSM Example: Synchronous Counter

The next transition

000 → 001 → 010 → 011 → 100 → 101 → 110 → 111
FSM Example: Synchronous Counter

The next transition

Diagram:

- States: 000, 001, 010, 011, 100, 101, 110, 111
- Transitions:
  - C/001 from 000 to 001
  - C/010 from 001 to 010
  - C/011 from 010 to 011
FSM Example: Synchronous Counter

The full transition set

```plaintext
000 → 001 C/001
001 → 010 C/010
010 → 011 C/011
011 → 100 C/100
100 → 101 C/101
101 → 110 C/110
110 → 111 C/111
111 → 000 C/000
```

Andrew H. Fagg: Embedded Real-Time Systems: FSMs
FSM Example: Synchronous Counter

Initial condition

- x/000
- C/000
- 000
- 001
- C/001
- 010
- C/010
- 011
- C/011
- 100
- C/100
- 101
- C/110
- 110
- C/111
- 111
- x/000
Example II: An Up/Down Counter

Suppose we have two events (instead of one): Up and down

• How does this change our state transition diagram?
Example II: An Up/Down Counter

From state 000, there are now two possible transitions:

- U/001: From state 000 to state 001
- D/111: From state 000 to state 111
Example II: An Up/Down Counter

Likewise for state 001…

000 → 001 → 010 → 011 → 101 → 100

000 → D/000 → U/010

001 → U/001 → D/111

010

011

100

101

110

111
Example II: An Up/Down Counter

The full transition set

- States: 000, 001, 010, 011, 100, 101, 110, 111
- Transitions:
  - Up: U/000 → U/001 → U/010 → U/011 → U/110 → U/111 → U/100
  - Down: D/000 → D/001 → D/010 → D/011 → D/110 → D/111

Diagram:
- Circles represent states.
- Arrows represent transitions with labels indicating the direction and the state.
FSMs and Control

How do we relate FSMs to Control?
• States are ?
FSMs and Control

How do we relate FSMs to Control?
• States are our memory of recent inputs

• Inputs are ?
FSMs and Control

How do we relate FSMs to Control?

• States are our memory of recent inputs

• Inputs are some processed representation of what the sensors are observing

• Outputs are ?
FSMs and Control

How do we relate FSMs to Control?

• States are our memory of recent inputs

• Inputs are some processed representation of what the sensors are observing

• Outputs are the control actions
FSMs: A Control Example

Suppose we have a vending machine:

• Accepts dimes and nickels
• Will dispense one of two things once $.20 has been entered: Jolt or Buzz Water
  – The “user” requests one of these by pressing a button
• Ignores select if < $.20 has been entered
• Immediately returns any coins above $.20
Vending Machine FSM

What are the states?
Vending Machine FSM

What are the states?

• $0
• $.05
• $.10
• $.15
• $.20
Vending Machine FSM

What are the inputs/events?
Vending Machine FSM

What are the inputs/events?

- Input nickel (N)
- Input dime (D)
- Select Jolt (J)
- Select Buzz Water (BW)
Vending Machine FSM

What are the outputs?
Vending Machine FSM

What are the outputs?

- Return nickel (RN)
- Return dime (RD)
- Dispense Jolt (DJ)
- Dispense Buzz Water (DBW)
- Nothing (Z)
Vending Machine Design

What is the initial state?
Vending Machine Design

What is the initial state?

- $S = 0$
Vending Machine Design

What can happen from $S = $0?

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<th>Event</th>
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</table>
Vending Machine Design

What can happen from $S = $0? 

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<thead>
<tr>
<th>Event</th>
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</thead>
<tbody>
<tr>
<td>N</td>
<td>$.05</td>
<td>Z</td>
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<td>D</td>
<td>$.10</td>
<td>Z</td>
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<td>J</td>
<td>$0</td>
<td>Z</td>
</tr>
<tr>
<td>BW</td>
<td>$0</td>
<td>Z</td>
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</table>

What does this part of the diagram look like?
Vending Machine Design

A piece of the state diagram:
Vending Machine Design

What can happen from $S = $0.05? 

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Vending Machine Design

What can happen from $S = \$0.05$?

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<th>Event</th>
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<tbody>
<tr>
<td>N</td>
<td>$$.10</td>
<td>Z</td>
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<td>D</td>
<td>$$.15</td>
<td>Z</td>
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<td>J</td>
<td>$$.05</td>
<td>Z</td>
</tr>
<tr>
<td>BW</td>
<td>$$.05</td>
<td>Z</td>
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</table>

What does the modified diagram look like?
Vending Machine Design

A piece of the state diagram:
Vending Machine Design

What can happen from $S = $0.10?

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</table>
### Vending Machine Design

What can happen from $S = \$0.10$?

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<td>$$.10</td>
<td>Z</td>
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<tr>
<td>BW</td>
<td>$$.10</td>
<td>Z</td>
</tr>
</tbody>
</table>
Vending Machine Design

A piece of the state diagram:

$0$

$0.05$

$0.10$

$0.15$

$0.20$
Vending Machine Design

What can happen from $S = \$0.15$?

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Vending Machine Design

What can happen from $S = $0.15? 

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</thead>
<tbody>
<tr>
<td>N</td>
<td>$.20</td>
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</tr>
<tr>
<td>D</td>
<td>$.20</td>
<td>RN</td>
</tr>
<tr>
<td>J</td>
<td>$.15</td>
<td>Z</td>
</tr>
<tr>
<td>BW</td>
<td>$.15</td>
<td>Z</td>
</tr>
</tbody>
</table>
Vending Machine Design

A piece of the state diagram:

\[ \text{\$0} \xrightarrow{\text{J/Z, BW/Z}} \text{\$0.05} \xrightarrow{\text{D/Z}} \text{\$0.10} \xrightarrow{\text{J/Z, BW/Z}} \text{\$0.15} \xrightarrow{\text{J/Z, BW/Z}} \text{\$0.20} \]

\[ \text{\$0} \xrightarrow{\text{x/Z}} \text{\$0} \xrightarrow{\text{N/Z}} \text{\$0.05} \xrightarrow{\text{N/Z}} \text{\$0.10} \xrightarrow{\text{D/RN}} \text{\$0.20} \]
Vending Machine Design

Finally: what can happen from S = $0.20?

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</table>
Finally, what can happen from \( S = \$0.20 \)?

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<tbody>
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<td>N</td>
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<td>RN</td>
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<tr>
<td>D</td>
<td>$.20</td>
<td>RD</td>
</tr>
<tr>
<td>J</td>
<td>$0</td>
<td>DJ</td>
</tr>
<tr>
<td>BW</td>
<td>$0</td>
<td>DBW</td>
</tr>
</tbody>
</table>
Vending Machine Design

The complete state diagram:

Andrew H. Fagg: Embedded Real-Time Systems: FSMs
A Robot Control Example

Consider the following task:

• The robot is to move toward the first beacon that it “sees”
• The robot searches for a beacon in the following order: right, left, front

What is the FSM representation?
Robot Control Example II

Consider the following task:
- The robot must lift off to some altitude
- Translate to some location
- Take pictures
- Return to base
- Land
- At any time: a detected failure should cause the craft to land

What is the FSM representation?
FSMs As Controllers

• Need code that translates sensory inputs into FSM events
• An FSM output can require an arbitrary amount of time
  – We will often implement this control action as a separate function call
• Control actions will not necessarily be fixed (but could be a function of sensory input)
FSMs As Controllers (cont)

- We might choose to leave some events out of the implementation
  - Only some events may be relevant to certain states
- When in a state, the FSM may also issue control actions (even when a new event has not arrived)
  - Again, this may be implemented as a function call
FSMs in C

```c
int state = 0;  // Initial state
while(1) {
    <do some processing of the sensory inputs>
    switch(state) {
        case 0:
            <handle state 0>
            break;
        case 1:
            <handle state 1>
            break;
        case 2: ...
    }
}
```
FSMs in C

```c
int state = 0;    // Initial state
while(1) {
    <do some processing of the sensory inputs>
    switch(state) {
        case 0:
            <handle state 0>
            break;
        case 1:
            <handle state 1>
            break;
        case 2: ...
    }
}
```

Variable declaration and initialization
FSMs in C

```c
int state = 0;   // Initial state
while(1) {
    <do some processing of the sensory inputs>
    switch(state) {
        case 0:
            <handle state 0>
            break;
        case 1:
            <handle state 1>
            break;
        case 2: ...
    }
}
```

A comment (use liberally)
FSMs in C

int state = 0; // Initial state
while(1) {
    <do some processing of the sensory inputs>
    switch(state) {
        case 0:
            <handle state 0>
            break;
        case 1:
            <handle state 1>
            break;
        case 2: ...  
    }
}
Loop forever
FSMs in C

```c
int state = 0; // Initial state
while(1) {
    <do some processing of the sensory inputs>
    switch(state) {
        case 0:
            <handle state 0>
            break;
        case 1:
            <handle state 1>
            break;
        case 2: ...
    }
}
```

"pseudo code": not really code, but indicates what is to be done
FSMs in C

```c
int state = 0; // Initial state
while(1) {
        <do some processing of the sensory inputs>
    switch(state) {
        case 0:
            <handle state 0>
            break;
        case 1:
            <handle state 1>
            break;
        case 2: ...
    }
}
```

In this case: we will translate the current sensory inputs into a representation of an event (if one has happened)
FSMs in C

```c
int state = 0; // Initial state
while(1) {
    <do some processing of the sensory inputs>
    switch(state) {
        case 0:
            <handle state 0>
            break;
        case 1:
            <handle state 1>
            break;
        case 2: ...
    }
}
```

Switch/case syntax allows us to cleanly perform many "if(x==y)" operations
FSMs in C

```c
int state = 0;  // Initial state
while(1) {
    <do some processing of the sensory inputs>
    switch(state) {
    case 0: 
        <handle state 0>
        break;
    case 1:
        <handle state 1>
        break;
    case 2: ...
    } 
}
```

If state==0, then execute the following code
FSMs in C

```c
int state = 0;  // Initial state
while(1) {
    <do some processing of the sensory inputs>
    switch(state) {
        case 0:  
            <handle state 0>
            break;
        case 1: 
            <handle state 1>
            break;
        case 2: ... 
    }
}

This code can be as complex as necessary
```
FSMs in C

```c
int state = 0;   // Initial state
while(1) {
    <do some processing of the sensory inputs>
    switch(state) {
        case 0:
            <handle state 0>
            break;  // break says to exit the switch (don't forget it or strange things can happen!)
        case 1:
            <handle state 1>
            break;
        case 2: ...
    }
}
```
FSMs in C

```c
int state = 0; // Initial state
while(1) {
    <do some processing of the sensory inputs>
    switch(state) {
        case 0:
            <handle state 0>
            break;
        case 1:
            <handle state 1>
            break;
        case 2: ...
    }
}
```

If state==1, then …
int state = 0;   // Initial state
while(1) {
    <do some processing of the sensory inputs>
    switch(state) {
        case 0:
            <handle state 0>
            break;
        case 1:
            <handle state 1>
            break;
        case 2: ...
    }
}
int state = 0; // Initial state
while(1) {
    <do some processing of the sensory inputs>
    switch(state) {
        case 0:
            <handle state 0>
            break;
        case 1:
            <handle state 1>
            break;
        case 2: ...
    }
}
FSMs in C (some other possibilities)

```c
int state = 0;  // Initial state
while(1) {
    <do some processing of the sensory inputs>
    switch(state) {
        case 0:
            <handle state 0>
            break;
        :
        default:
            <handle default case>
            break;
    }
    <do some low-level control>
}
```
FSMs in C (some other possibilities)

```c
int state = 0;  // Initial state
while(1) {
    <do some processing of the sensory inputs>
    switch(state) {
        case 0:
            <handle state 0>
            break;
        ...
        default:
            <handle default case>
            break;
    }
    <do some low-level control>
}

Matches any state (if we reach this point)
```
FSMs in C (some other possibilities)

```c
int state = 0; // Initial state
while(1) {
    <do some processing of the sensory inputs>
    switch(state) {
        case 0:
            <handle state 0>
            break;
        
        default:
            <handle default case>
            break;
    }
    <do some low-level control>
}
(possibly) alter some control outputs (e.g., steering direction)
```
case STATE_10cents:
    // $.10 has already been deposited
    switch (event) {
        case EVENT_NICKEL:  // Nickel
            state = STATE_15cents;  // Transition to $.15
            break;
        case EVENT_DIME:     // Dime
            state = STATE_20cents;  // Transition to $.2
            break;
        case EVENT_JOLT:     // Select Jolt
        case EVENT_BUZZ:     // Select Buzzwater
            display_NOT_ENOUGH();
            break;
        case EVENT_NONE:     // No event
            break;  // Do nothing
    }
break;
FSMs in C: Processing for Individual States

case STATE_10cents:
    // $.10 has already been deposited
    switch(event) {
        case EVENT_NICKEL: // Nickel
            state = STATE_15cents; // Transition to $.15
            break;
        case EVENT_DIME: // Dime
            state = STATE_20cents; // Transition to $.2
            break;
        case EVENT_JOLT: // Select Jolt
        case EVENT_BUZZ: // Select Buzzwater
            display_NOT_ENOUGH();
            break;

        case EVENT_NONE: // No event
            break; // Do nothing
    }

};
break;
FSMs in C: Processing for Individual States

case STATE_10cents:
    // $.10 has already been deposited
    switch (event) {
        case EVENT_NICKEL:  // Nickel
            state = STATE_15cents;  // Transition to $.15
            break;
        case EVENT_DIME:  // Dime
            state = STATE_20cents;  // Transition to $.2
            break;
        case EVENT_JOLT:  // Select Jolt
        case EVENT_BUZZ:  // Select Buzzwater
            display_NOT_ENOUGH();
            break;
        case EVENT_NONE:  // No event
            break;  // Do nothing
    }
    break;

A nickel has been received
FSMs in C: Processing for Individual States

case STATE_10cents:
    // $.10 has already been deposited
    switch(event) {
    case EVENT_NICKEL:   // Nickel
        state = STATE_15cents;  // Transition to $.15
        break;
    case EVENT_DIME:     // Dime
data = STATE_20cents;  // Transition to $.2
        break;
    case EVENT_JOLT:     // Select Jolt
    case EVENT_BUZZ:     // Select Buzzwater
        display_NOT_ENOUGH();
        break;

    case EVENT_NONE:     // No event
        break;             // Do nothing

    "};
    break;

Change state for next iteration of the while() loop
case STATE_10cents:
    // $.10 has already been deposited
    switch(event) {
        case EVENT_NICKEL:     // Nickel
            state = STATE_15cents;  // Transition to $.15
            break;
        case EVENT_DIME:       // Dime
            state = STATE_20cents;  // Transition to $.2
            break;
        case EVENT_JOLT:       // Select Jolt
        case EVENT_BUZZ:       // Select Buzzwater
            display_NOT_ENOUGH();
            break;
        case EVENT_NONE:       // No event
            break;               // Do nothing
    }
    break;
}
A Note on “Style” in C

• The numbers that we assigned to the different states are arbitrary (and at first glance, hard to interpret)
• Instead, we can define constant strings that have some meaning

• Replace: 0, 1, 2, 3, 4, 5
• With: STATE_00, STATE_05, STATE_10, STATE_15, STATE_20
A Note on “Style” in C

In C, this is done by adding some definitions to the beginning of your program (either in the .c file or the .h file):

```c
#define STATE_00   0
#define STATE_05   1
#define STATE_10   2
#define STATE_15   3
#define STATE_20   4
```