

Fast and Precise Power Prediction for Combinational Circuits

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Abstract

The power consumed by a combinational circuit is dictated by the switching activities of all signals associated with the circuit. An analytical approach is proposed for calculating signal activities for combinational circuits. The approach is based on a Markov chain signal model, and directly accounts for correlations present among the signals. The accuracy of the approach is verified by comparing signal activity values calculated using the proposed approach with corresponding values produced through simulation studies. It is also demonstrated that the proposed approach is computationally efficient.

1. Introduction

Power consumption of integrated circuits (ICs) is of growing concern as more electronic devices are being deployed in mobile and portable applications, e.g., PDAs, mobile telephones, and other battery-powered electronic devices. As the functionality of such devices increases, so does the complexity and sophistication of the underlying circuits. More complexity and faster clock rates generally translate into higher power consumption for a given hardware implementation technology. Because battery technology has not improved at the same rate as IC technology, there is strong motivation to design circuits that are as power efficient as possible to extend battery life for portable devices.

The focus of this paper is the development of an analytical tool for predicting power consumption of combinational circuits. This tool, which is implemented in software, can be utilized during the design phase to give the designer quick and accurate predictions of power consumption for a given circuit design.

Several similar and related approaches to this problem have been proposed in the past, including

simulation-based [1] and analytical approaches [2, 3, 4]. A good survey of past approaches can be found in [5]. Generally, simulation-based approaches achieve high accuracy but require long execution times; in contrast, the analytical approaches are faster but are generally less accurate. In this paper a new analytical approach is proposed that achieves fast execution time and accuracy that is comparable with simulation-based methods. As explained below, the particular focus is on power consumption of circuits implemented in CMOS, but the proposed approach may be applicable for other technologies as well.

Power consumption in a CMOS circuit is primarily due to three types of current flow: leakage current, switching transient current, and load capacitance charging current [9]. The last is the dominant component of power consumption in CMOS devices, and is strongly dependent on signal switching activity.

Let S denote the set of all signals associated with a circuit. For each $s \in S$, let $C(s)$ denote the capacitive load associated with signal s . Also, let $\alpha(s)$ denote the activity of signal s , which has a value between zero and one, and represents the signal's normalized average frequency relative to the frequency of a system clock, f . Thus, $f\alpha(s)$ gives the average frequency of signal s . Based on these assumptions and notation, the average power for a CMOS circuit operating at a voltage level of V can be expressed as [4, 5]:

$$\text{Power}_{\text{avg}} = \frac{1}{2} V^2 f \sum_{s \in S} C(s) \alpha(s). \quad (1)$$

The problem addressed in this paper is to determine the activity of all signals of a combinational circuit given an appropriate probabilistic model for the primary input signals that drive the circuit. The signal model proposed in this paper is based on a Markov chain. The signal activity is easily computed from the parameters associated with the proposed signal model. In the proposed approach, signals with known Markov chain representations are propagated through the circuit to produce Markov chain representations for the outputs of all gates in the circuit. Accuracy of the approach is

verified by comparing signal activities produced by the proposed method with corresponding activities produced through simulation studies. When compared with other related approaches, a key aspect of the proposed approach is that correlations present among the signals due to re-convergent fan-out are accounted for directly.

2. Previous Related Approaches

2.1 Signal Probability Calculation

In [2], probabilistic signal modeling for combinational circuits was first introduced. Each signal is modeled with a single probabilistic parameter that defines the probability of a signal having a logical value of one. For signal x , the probability that x has logic value 1 is defined by $P(x) = P(x=1)$. Two algorithms for calculating signal probabilities are introduced in [2]. These approaches require that a Boolean function expression associated with each signal be derived in terms of the primary inputs. Because the number of terms in these expressions can grow exponentially with the number of inputs, the complexity of these approaches can be prohibitive for practical circuits.

A computationally efficient algorithm for calculating signal probabilities is introduced in [7], named "Algorithm 1," which operates by propagating probability values through the gates of circuit, thereby drastically reducing the size of the Boolean functions that must be evaluated. This algorithm is simple and fast – it has a linear complexity in the number of gates – but is not accurate for all classes of circuits.

Another algorithm is proposed in [7] called the Weighted Averaging Algorithm (WAA), which generally achieves better accuracy than Algorithm 1 and has a comparable time complexity. However, the WAA still does not always produce correct values.

A method for accounting for signal probability correlations was developed in [6] named the correlation coefficient method (CCM). By using this approach, the probability of the output of a two-input gate can be more accurately calculated, given the probabilities of the two inputs and an associated correlation factor associated with the two signals. In this algorithm, the correlation factor can also be calculated analytically by means of a set of basic propagation rules.

2.2. Signal Activity Calculation

The above-described approaches of [2], [6], and [7] are concerned with determining the probabilities of signal values, not the probabilities of signal transitions, i.e., activities, which are necessary for estimating power

consumption, refer to Eq. 1. An early approach for estimating signal activities was developed in [3], in which signals of a circuit are modeled to be mutually independent strict-sense-stationary (SSS) mean-ergodic 0-1 processes. Under these assumptions, the activity of a signal y from a circuit with n -primary inputs can be expressed as

$$\alpha(y) = \sum_{i=1}^n P\left(\frac{\partial y}{\partial x_i}\right) \alpha(x_i) \quad (2)$$

where $\partial y / \partial x_i$ is the Boolean difference of function y with respect to x_i and is defined by

$$\begin{aligned} \frac{\partial y}{\partial x_i} = & y|_{x_i=1} \oplus y|_{x_i=0} = y(x_1, \dots, x_{i-1}, 1, x_{i+1}, \dots, x_n) \\ & \oplus y(x_1, \dots, x_{i-1}, 0, x_{i+1}, \dots, x_n). \end{aligned} \quad (3)$$

Intuitively, the Boolean difference $\partial y / \partial x_i$ defines whether a transition of signal x_i will cause a transition in output signal y . Specifically, if the Boolean difference function evaluates to one, then a transition of signal x_i causes a transition in y . So, the probability of the Boolean difference function, $P\left(\frac{\partial y}{\partial x_i}\right)$, defines the probability that a change in y will occur given that there is a change in x_i .

The calculation of the probability of the Boolean difference terms, i.e., $P\left(\frac{\partial y}{\partial x_i}\right)$, this calculation can be

complicated for large and complex circuits. In [3], the calculation of these terms is accomplished by first representing the nodes of the circuit with a binary decision diagram (BDD) [3, 5]. In practice, the BDD approach often achieves linear or near linear time complexity; however, in the worst case the complexity can grow exponentially with the number of gates.

It is noted in [4] that Eq. 2, i.e., the approach described in [3], fails to consider the effect of simultaneous switching of gate inputs. Each Boolean difference term associated with Eq. 2 describes an input-switching event in which exactly one of the inputs makes a transition. Thus, Eq. 2 does not account for events involving simultaneous switching of two or more of the input signals. The concept of the generalized Boolean difference was introduced in [4] to account for simultaneous switching, and is denoted as follows:

$$\begin{aligned} \frac{\partial y^k}{\partial x_{i_1} \partial x_{i_2} \dots \partial x_{i_k}} = & (y|_{x_{i_1}=b_{i_1}, x_{i_2}=b_{i_2}, \dots, x_{i_k}=b_{i_k}}) \\ & \oplus (y|_{x_{i_1}=\overline{b_{i_1}}, x_{i_2}=\overline{b_{i_2}}, \dots, x_{i_k}=\overline{b_{i_k}}}), \end{aligned} \quad (4)$$

where k is a positive integer, x_{i_j} , $j=1,2,\dots,k$, are distinct mutually independent primary inputs of y , and

b_{i_j} are binary values of 0 or 1. Note that if the generalized Boolean difference evaluates to one, then the simultaneous transitions of signals $(x_{i_1}, x_{i_2}, \dots, x_{i_k})$ from $(\overline{b_{i_1}}, \overline{b_{i_2}}, \dots, \overline{b_{i_k}})$ to $(\overline{b_{i_1}}, \overline{b_{i_2}}, \dots, \overline{b_{i_k}})$ or from $(\overline{b_{i_1}}, \overline{b_{i_2}}, \dots, \overline{b_{i_k}})$ to $(b_{i_1}, b_{i_2}, \dots, b_{i_k})$ will cause a transition at y .

Eq. 2 is adapted in [4] using the generalized Boolean difference concept to account for simultaneous switching, resulting in:

$$\alpha(y) = \sum_{i=1}^n P_C \left(\frac{\partial y}{\partial x_i} \right) \left(\alpha(x_i) \prod_{\substack{j=1 \\ j \neq i}}^n [1 - \alpha(x_j)] \right) \quad (5)$$

$$+ \frac{1}{2} \left\{ \sum \left[P_C \left(\frac{\partial^2 y}{\partial x_i \partial x_j} \right) + P_C \left(\frac{\partial^2 y}{\partial x_j \partial x_i} \right) \right] \left(\alpha(x_i) \alpha(x_j) \prod_{j \in \{1, 2, \dots, n\} - \{i, j\}} [1 - \alpha(x_j)] \right) \right\} + \dots$$

$$+ \frac{1}{2^{n-1}} \left[P_C \left(\frac{\partial^n y}{\partial x_1 \partial x_2 \dots \partial x_n} \right) + P_C \left(\frac{\partial^n y}{\partial x_1 \partial x_2 \dots \partial x_n} \right) + \dots + P_C \left(\frac{\partial^n y}{\partial x_1 \partial x_2 \dots \partial x_n} \right) \right] \left(\prod_{i=1}^n \alpha(x_i) \right)$$

where $P_C \left(\frac{\partial y}{\partial x_i} \right)$, $P_C \left(\frac{\partial^2 y}{\partial x_i \partial x_j} \right)$, \dots , $P_C \left(\frac{\partial^n y}{\partial x_1 \partial x_2 \dots \partial x_n} \right)$ are

conditional probabilities of the generalized Boolean differences under the condition that only the indicated inputs simultaneously switch, and the rest do not. Details on how to calculate these conditional probabilities can be found in [4].

The approaches of [3] and [4] can have high computational complexities because the number of terms in the underlying equations/transformations can grow exponentially with the number of primary inputs to the circuit. Trade-offs between computational complexity and accuracy are possible relative to the evaluation of Eq. 2 and Eq. 5 (associated with [3] and [4], respectively). Instead of deriving a signal's logic function in terms of the circuit's primary inputs, the parameters to the immediate inputs of the signal's logic gate can be used. This type of "gate-by-gate" technique will generally introduce error because it does not account for correlations present among the internal signals that drive the gates within the circuit.

3. Markov Chain Signal Model

3.1. Preliminaries

In this section we introduce a signal model that is based on a Markov chain having three event parameters. It is shown that the proposed Markov chain model is equivalent to the two-parameter probability/activity signal model of [3] and [4]. The advantage of modeling signals with Markov chains is that it makes it possible to compute correlations between signals related to both probability and activity.

The approach derived here can be viewed as a generalization of the approach in [6]. Instead of tracking

a correlation factor for the single probability parameter model, transformations for correlation factors associated with the three parameters of the Markov model are derived. This ultimately leads to a fast and accurate "gate-by-gate" algorithm for calculating signal probabilities and activities.

As illustrated in Figure 1, the proposed Markov chain signal model has three event parameters for signal A . The event denoted by A represents the signal being in state 1, and A_1 and A_2 represent the events that there is a transition from state 0 to 1 and from state 1 to 0, respectively. Note that the probability of event A is denoted by $P(A)$, and is equivalent to the signal probability defined in the previous section.

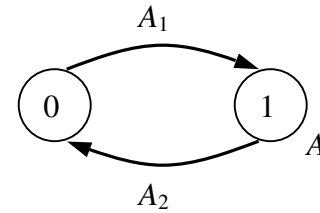


Figure 1. Proposed Markov chain signal model.

For notational convenience and clarity, we will denote the value of $P(A)$ as p_A (for the value of the probability of signal A) and the value of the activity $\alpha(A)$ as α_A (for the value of the activity of signal A) throughout the rest of the paper. Using these notations and applying basic properties of Markov chains along with the definition of signal activity, the following expressions can be derived for $P(A)$, $P(A_1)$ and $P(A_2)$:

$$P(A) = p_A, \quad P(A_1) = \frac{\alpha_A}{2(1 - p_A)}, \quad P(A_2) = \frac{\alpha_A}{2p_A}. \quad (6)$$

Thus, if the values of both the probability and activity parameters of a signal are known (i.e., p_A and α_A), then the probabilities of the three events associated with the proposed Markov model for the signal are completely determined. Likewise, knowing the probability values of the three parameters of the Markov model fully determines the probability and activity parameters of the signal.

In order to define correlations between two signals modeled with Markov chains, some basic definitions are needed. Let A and B denote two events and let $P(AB)$ denote the probability of both A and B occurring. From basic probability theory [8], $P(AB) = P(A/B)P(B)$, where $P(A/B)$ represents the probability of A given B . Also, the correlation coefficient of two events A and B is defined as

$$\rho_{AB} = \frac{\sigma_{AB}}{\sigma_A \sigma_B} \quad (7)$$

where σ_{AB} is the covariance and σ_A and σ_B are the positive square roots of the variances of A and B . It can be shown that

$$\rho_{AB} = \frac{P(AB) - P(A)P(B)}{\sqrt{P(A)(1-P(A))}\sqrt{P(B)(1-P(B))}}. \quad (8)$$

In order to simplify later derivations, it is convenient to define the correlation factor C_{AB} of two events A and B as

$$C_{AB} = \frac{P(AB)}{P(A)P(B)} = \frac{P(A/B)}{P(A)} = \frac{P(B/A)}{P(B)}. \quad (9)$$

By applying Eq. 8 to Eq. 9, the following relationship can be derived:

$$\rho_{AB} = \frac{P(A)}{\sqrt{P(A)(1-P(A))}} \frac{P(B)}{\sqrt{P(B)(1-P(B))}} (C_{AB} - 1). \quad (10)$$

Thus, C_{AB} is related to ρ_{AB} through scaling and shifting. The value of ρ_{AB} , by definition [8], is a real number in the interval $[-1, 1]$; therefore, according to Eq. 10, C_{AB} takes on real non-negative values. Also, $\rho_{AB} = 0$ corresponds to $C_{AB} = 1$, and indicates that the events A and B are mutually independent. Similarly, $\rho_{AB} < 0$ (i.e., A and B are negatively correlated) corresponds to $0 \leq C_{AB} < 1$, and $\rho_{AB} > 0$ (i.e., A and B are positively correlated) corresponds to $C_{AB} > 1$.

3.2. Markov Chain Model for Basic Logic Gates

The focus in this subsection is on deriving the Markov chain model for the output of a basic logic gate in which the Markov chain models of the input signals are known. The simple case of a NOT gate is considered first followed by the analysis of two-input basic logic gates.

For a NOT gate with input A , the Boolean output function is given by $Y = \bar{A}$. From Figure 1, it is clear that the Markov model for Y is given by

$$P(Y) = 1 - P(A), \quad P(Y_1) = P(A_2), \quad P(Y_2) = P(A_1). \quad (11)$$

Consider now the case of a two-input basic logic gate. Assuming the Markov chain models of inputs A and B are known, the objective is to derive the Markov chain model for output signal Y . A key to deriving the Markov chain model for signal Y is to represent the state transition diagram associated with the gate's two inputs, as shown in Figure 2. The four states in the figure correspond to the four input combinations for the two

inputs. The first digit of each state label corresponds to the value of A , and the second to the value of B , e.g., the state labeled "01" corresponds to $A = 0$ and $B = 1$. Although not labeled on the figure, the directed edges represent transition events. To illustrate the notation to label transition events, "00→10" will be used to represent the event that input signal A transitions from 0 to 1 and signal B stays in state 0.

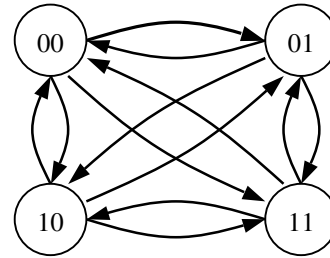


Figure 2. State transition diagram for a two-input gate.

The known parameters of the Markov chain models for signals A and B are given by $P(A)$, $P(A_1)$, $P(A_2)$, $P(B)$, $P(B_1)$, and $P(B_2)$. Also assumed to be known are the correlation factors for pairs of events associated with the Markov chain models for the inputs. From Eq. 9 note that $P(AB) = P(A)P(B)C_{AB}$, where C_{AB} is the correlation factor associated with events A and B . Similarly, the correlation factor $C_{A_1B_2}$ enables the calculation of $P(A_1B_2)$ using the fact that $P(A_1B_2) = P(A_1)P(B_2)C_{A_1B_2}$. Recall from Eq. 10 that independent events correspond to a correlation factor of unity. Given the Markov chain models for signals A and B (and the corresponding correlation factors) it is possible to derive the probability associated with every event shown in the state transition diagram of Figure 2. A complete tabulation of these expressions can be found in [11].

Deriving a Markov chain model for the output (Y) of a two-input gate depends on the particular function of the gate. To illustrate, consider the specific example of an AND gate, i.e., $Y = AB$. For an AND gate, the output takes on logic value 1 if and only if both inputs are 1. Thus,

$$P(Y) = P(11) = P_A P_B C_{AB}. \quad (12)$$

The event Y_1 is associated with three events from Figure 2, namely: 00→11, 01→11, and 10→11. Thus, equality can be established as follows:

$$P(\bar{Y})P(Y_1) = P(00)P(00 \rightarrow 11) + P(01)P(00 \rightarrow 11) + P(01)P(00 \rightarrow 11). \quad (13)$$

Solving Eq. 13 for $P(Y_1)$ and using Eqs. 6 results in the following expression:

$$P(Y_1) = \left(\frac{1}{2} \lambda_A p_B \alpha_A + \frac{1}{2} \lambda_B p_A \alpha_B \right) / (1 - p_A p_B C_{AB}) - \left[\frac{1}{4} (\lambda_A C_{A_1 B_2} + \lambda_B C_{A_2 B_1} - \lambda C_{A_1 B_1}) \alpha_A \alpha_B / (1 - p_A p_B C_{AB}) \right] \quad (14)$$

The parameters λ , λ_A , and λ_B are simply functions of probabilities and correlations factors and are used for notational convenience; expressions for these parameters can be found in [11]. Derivation for $P(Y_2)$ follows in a similar fashion and can be expressed as

$$P(Y_2) = \frac{\alpha_1}{2p_1} + \frac{\alpha_2}{2p_2} - \frac{\alpha_1 \alpha_2}{2p_1 2p_2} C_{A_2 B_2}. \quad (15)$$

Derivations of $P(Y)$, $P(Y_1)$, and $P(Y_2)$ for two-input OR and XOR gates are included in [11]. Methods for calculating/propagating correlation factors through basic elements of a circuit are also included in [11].

4. Markov Chain Propagation Algorithm

This section describes a proposed Markov Chain Propagation (MCP) algorithm for determining the Markov chain models for all signals of a given combinational circuit. The Markov chain signal model of Section 3 is employed, and it is assumed that the parameters of the model are known for the circuit's primary inputs. The overall approach is to propagate signal information associated with the Markov chain model through the circuit in a "gate-by-gate" fashion. Recall that once the Markov chain model is determined for all signals, the signal activities and circuit power estimate are determined using Eq. 6 and Eq. 1, respectively. It is assumed that the given circuit is specified at the level of basic logic gates.

MCP Algorithm

Step 1: Represent the given combinational circuit as a directed acyclic graph (DAG).

Vertices of the DAG correspond to basic gates and edges represent signals. Two extra vertices (a source and a sink) are included in the DAG to accommodate the primary inputs and outputs of the circuit. An example of how to represent a circuit with the DAG model is illustrated by Figures 3(a) and 3(b).

Step 2: Perform a topological sort [10] on the DAG to obtain an ordering of the gates.

See Figure 3(c).

Step 3: Transform to two-input basic logic gates.

As shown in Figure 3(d), replace all basic gates having more than two inputs with an equivalent sequence of two-input basic gates.

Step 4 Partition the circuit into levels.

As shown in Figure 3(e), levels are defined at the input and output of each basic gate. Note that there is at most one gate between any two consecutive levels.

Step 5: Successively apply propagation rules at each level.

Apply the propagation rules from [11] for calculating the parameters of the Markov model for the basic gate outputs and the associated correlation factors.

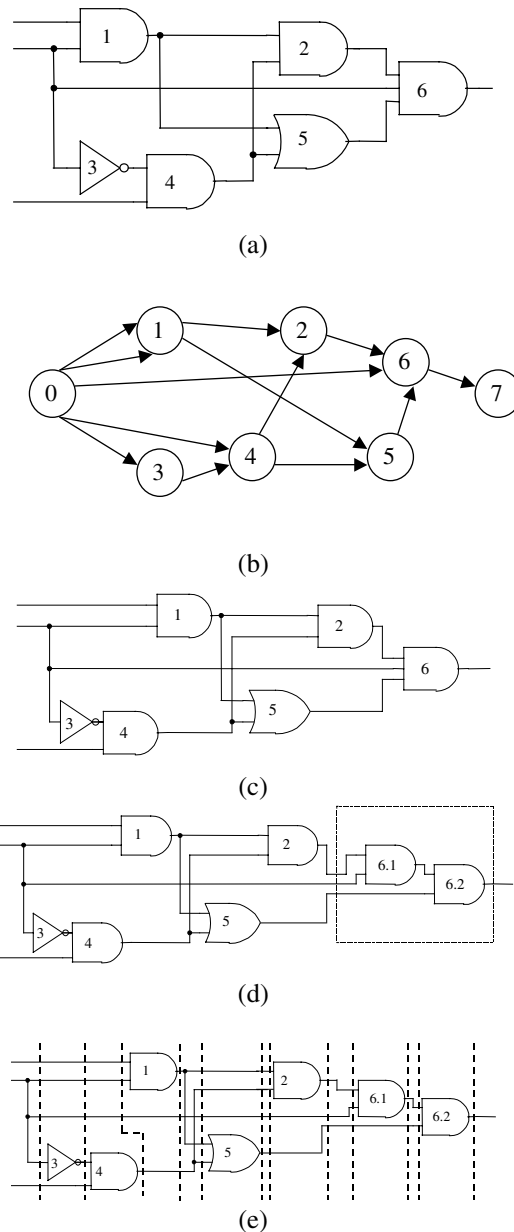


Figure 3. Illustration of the MCP Algorithm.

For a circuit with M signals and N gates, the time complexity of the MCP Algorithm can be shown to be $\Theta(M + N)$. Due to space limitations, a detailed derivation of the time complexity of the MCP Algorithm is not included here, but can be found in [11].

5. Experimental Results

The MCP Algorithm has been implemented and evaluated using several test circuits. To verify the accuracy of the results produced by the MCP algorithm, PSpice[®] circuit simulations were performed on the same test circuits. In the simulation studies, time-series realizations from the assumed Markov chain model for each primary input were used to drive the circuit simulation. Estimates of signal probabilities were derived from the simulations by counting the fraction of time each signal took on a value of unity. Estimates of signal activities were derived from the simulations by counting signal transitions.

The MCP Algorithm was also evaluated using a circuit named C432 from the ISCAS-85 Benchmark Set. For this circuit there are a total of 145 distinct signals, not including the primary inputs. (Note that there are a total of 432 physical signals, which includes fan-out signals.) Table 1 shows the distribution of absolute differences and relative percentage errors between activity values computed by the MCP Algorithm and those derived through simulation. Other circuits were also tested and these results also indicate the accuracy of the MCP Algorithm.

Table 1. Accuracy for the MCP Algorithm.

Absolute Diff. Range	Number of Signals	Relative Error Range (%)	Number of Signals
[0, 0.01]	70	[0, 1]	43
(0.01, 0.02]	35	(1, 2]	41
(0.02, 0.03]	19	(2, 5]	31
(0.03, 0.04]	10	(5, 10]	25
(0.04, 0.05]	10	(10, 20]	3
(0.05, 0.06]	1	(20, 50]	2
(0.06, 1]	0	>50	0

6. Summary and Future Work

The problem of determining the activities of all signals of a combinational circuit is addressed in this paper. A new signal model is proposed based on a Markov chain. Signal activity is easily computed from the parameters associated with the proposed signal model. In the proposed approach, signals with known Markov chain representations are propagated through the

circuit to produce a Markov chain representation for the output of each gate in the circuit. Accuracy of the approach is verified by comparing signal activities produced by the proposed method with corresponding activities produced through simulation studies. These initial testing results will be extended in future work by testing more and larger circuits.

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