

Planning Spatial Paths for Automated Spray Coating Applications

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Abstract

Automated spray coating is an important component of finishing operations in many industries. Robotic manipulators are used in such units to traverse around the object to be coated. This paper describes a framework for determining a good spatial path for the manipulator, in order to achieve uniform deposition over the surface and reduce wastage of coating material outside the surface. By imposing an arc-length based conditioning, the formulated variational problem is shown to be well-posed. The variation in deposition rate, the deposition outside the surface, and the arc length are minimized in a weighted combination. The necessary conditions for a minimizing spatial path are given by a set of nonlinear differential equations that are numerically solved.

1 Introduction

This paper develops an analytical framework for choosing spatial paths and velocities for applicators used in automated spray coating. The discussion focuses on finishing applications in manufacturing, where the objective is to improve the external appearance of the surface through the application of paints/polishes.

In many cases, the perceived quality of the manufactured product often contributes significantly to the sale of the product. The automobile industry is a typical example [2].

In order to achieve a high repeatability of a specified degree of performance such as achieving a uniform coat, robotic manipulators are used as spray coating applicators in many plants. With the availability of these manipulators, the traversal of the spray gun over the surface can be accurately controlled. The actual spatial path and velocity of the applicator can be planned in such a way as to ensure a uniform coating over the entire surface. Once the paths and velocities are programmed, the repeatability of robotic manipulators ensures a consistent product quality. Further, the paths can also be planned to

ensure minimum wastage of coating material, maximum transfer efficiency and less over-spray, which not only reduces coating costs, but also provides for a cleaner environment. Case studies of economic savings are presented in industrial finishing literature. In one such case, off line simulation studies on the coating process have helped realize a saving of 11,000 dollars per day in a truck manufacturing facility [1].

With such advantages to be gained, proper planning of such spatial paths and velocities thus becomes important. One method to ensure proper selection of paths and velocities is to develop a mathematical framework and formulate the path selection problem as a constrained optimization problem (e.g., see [3, 6, 7, 8]).

The problem of determining improved spatial paths for achieving uniform deposition with lesser energy expenditures is discussed in [6]. However, an analytical framework with sufficient detail is not provided, and the discretization of the problem at the beginning renders it a computationally expensive approach. An analytical framework for the problem is provided in [3], but the discussion is restricted to the case where the spatial path for the applicator is specified, and the velocity profile is to be determined. Faster solution techniques for the approach in [3] are developed in [7]. This paper modifies the formulation in [3, 7] to include spatial and temporal path planning (i.e., determining the spatial path and the velocity of the applicator). Further, the scope of the work is enlarged to minimize wastage of coating material outside the surface. Thus, a framework for achieving the twofold objectives of uniform deposition and good transfer efficiency is provided.

Computing a spatial path that achieves these objectives analytically demands the knowledge of a deposition rate function that provides a relationship between the spatial location of the applicator (with the spray gun) and film accumulation on the surface. Thus, given a point s on the surface \mathcal{S} , and a path $\mathbf{p}(t)$ specifying the position and orientation of the applicator at time t , it is assumed that the rate of film accumulation at s is specified by a known func-

¹This work was supported by the National Science Foundation under grant 8803017-ECD to the Engineering Research Center for Intelligent Manufacturing Systems.

tion $f(\mathbf{s}, \mathbf{p}(t))$. The availability of an analytical form for this film accumulation rate function is assumed. In the event that only empirical representations are available, standard distributions such as a bivariate Cauchy distribution or a bivariate Gaussian distribution can be used to model the rate of film accumulation, as done in [3, 6]. The framework proposed in this paper can be incorporated in simulation tools for off-line studies of the spray coating process (e.g., see [8]).

The organization of the paper is as follows. Section 2 presents some basic assumptions and definitions. Fundamental quantities such as film thickness and average thickness are defined. Two performance indices are introduced for uniform deposition—the variation in film thickness, which has been used in [3, 7] and the variation in deposition rate, with which an easier analytical formulation is made possible. A transfer efficiency index is introduced for quantifying the wastage of material outside the coated surface, relative to the amount of material coated on the surface. Section 3 outlines a solution to the problem of achieving a uniform deposition, and introduces regularization of the indices in order to make the problem well-posed. Numerical studies are outlined in Section 4, where, necessary conditions are derived for the spatial path, for coating a flat panel with a specified model of film accumulation. The effect of considering the transfer efficiency cost is demonstrated for the same example. Results of the simulation studies are presented. The conclusions are outlined in Section 5.

2 Basic Assumptions and Definitions

Consider a spray coating applicator used for coating a surface that is described by a set of points $\mathbf{s} \in \mathcal{S}$. Let the position and orientation of the applicator be specified by a six-dimensional vector of coordinate positions and orientations, $\mathbf{p} \in \mathcal{R}^6$, given by $\mathbf{p} = [p_x, p_y, p_z, p_\phi, p_\psi, p_\theta]$. In the event that the applicator is in a continuous state of motion, the coordinates are functions of time, and are represented by a set of time functions $\mathbf{p}(t) = [p_x(t), p_y(t), p_z(t), p_\phi(t), p_\psi(t), p_\theta(t)]$. Given this vector of positions and orientations, it is assumed that a deposition rate function, characterized by a mapping $f : \mathcal{S} \times \mathcal{R}^6 \rightarrow \mathcal{R}^+$ is specified. This mapping characterizes the rate of deposition of film onto a point on the surface, given a specific position and orientation of the applicator. Given this instantaneous rate of film accumulation, the total film accumulated at a point $\mathbf{s} \in \mathcal{S}$ over a time interval $[0, T]$ is given by the function $F(\mathbf{s}, \mathbf{p}(\cdot), T)$ as

$$F(\mathbf{s}, \mathbf{p}(\cdot), T) = \int_0^T f(\mathbf{s}, \mathbf{p}(t)) dt. \quad (1)$$

This expression gives the total film accumulation for each point on the surface. An expression for the average film accumulation over the entire surface, given by $G(\mathbf{p}(\cdot), T)$ can be formulated as

$$G(\mathbf{p}(\cdot), T) = \frac{1}{\mathcal{A}_S} \int_{\mathcal{S}} F(\mathbf{s}, \mathbf{p}(\cdot), T) ds, \quad (2)$$

where,

$$\mathcal{A}_S = \int_{\mathcal{S}} ds. \quad (3)$$

2.1 Indices for Uniform Deposition

A standard measure that has been adopted in the literature (e.g., [3, 7]) to represent uniformity of film coat is the variation in film thickness, $\mathcal{V}(\mathbf{p}(\cdot), T)$, defined as

$$\mathcal{V}(\mathbf{p}(\cdot), T) = \frac{1}{\mathcal{A}_S} \int_{\mathcal{S}} (F(\mathbf{s}, \mathbf{p}(\cdot), T) - F_d)^2 ds, \quad (4)$$

where F_d is a desired thickness value. While this measure represents the deviation in film thickness, an alternate measure that is used in the problems considered in this paper is the variation in deposition rate, $\mathcal{D}(\mathbf{p}(\cdot), T)$ defined as

$$\mathcal{D}(\mathbf{p}(\cdot), T) = \int_0^T \left[\int_{\mathcal{S}} (f(\mathbf{s}, \mathbf{p}(t)) - f_d)^2 ds \right] dt. \quad (5)$$

This measure aims to describe the deviation from a desired deposition rate f_d over the surface, over the time interval $[0, T]$.

2.2 The Transfer Efficiency Index

A common problem in coating applications is the excessive wastage of coating material that occurs while coating a surface. For achieving uniform deposition, it may be necessary to traverse slowly over areas of the surface that are likely to have a lower deposition rate (such as edges and bends). This may result in a significant wastage of coating material. To minimize this loss, the total film accumulation on the surface is added to the cost function. Thus, if the cost function for uniform deposition (variation in film thickness or variation in deposition rate) is denoted by $\mathcal{C}(\mathbf{p}(\cdot), T)$, a new cost function $\mathcal{C}_a(\mathbf{p}(\cdot), T)$ can be defined as

$$\mathcal{C}_a(\mathbf{p}(\cdot), T) = \mathcal{C}(\mathbf{p}(\cdot), T) - \kappa \mathcal{A}_S G(\mathbf{p}(\cdot), T), \kappa > 0. \quad (6)$$

The parameter κ acts as a weight for indicating the relative importance of total film accumulation in the cost function. Thus, increasing κ increases the importance of achieving good transfer efficiency.

The motivation for including this particular term can be explained. given that, along a specific path,

the total amount of coating material released from the nozzle is constant, the objective is to minimize the amount of coating material falling outside the surface. This is the same as maximizing the amount of coating material incident on the surface, which is given by the average thickness times the area of the surface. By varying the parameter κ , the amount of material wastage can thereby be controlled.

3 The Uniform Deposition Problem

In this section, solution techniques are outlined for determining a path $\mathbf{p}(\cdot)$ that achieves uniform deposition. Two indices were proposed in the previous section, that reflected the uniformity of film accumulation over the surface. In order to achieve uniform deposition over the surface \mathcal{S} , both measures, i.e., $\mathcal{V}(\mathbf{p}(\cdot), T)$, and $\mathcal{D}(\mathbf{p}(\cdot), T)$ can be minimized by appropriately choosing the path $\mathbf{p}(\cdot)$. While the minimization of $\mathcal{V}(\mathbf{p}(\cdot), T)$ is considered in [3, 7], it is shown that minimizing $\mathcal{D}(\mathbf{p}(\cdot), T)$ leads to a simpler analytical formulation.

3.1 Necessary Conditions for a Minimizing $\mathbf{p}(\cdot)$

The necessary conditions for minimizing $\mathcal{V}(\mathbf{p}(\cdot), T)$ or $\mathcal{D}(\mathbf{p}(\cdot), T)$ are provided by the Euler-Lagrange equations in the calculus of variations (e.g., [4]). The variational formulation, however, requires that the cost function be an integral over time and that the integrand depend only on evaluations at each time instant. This requirement enforces the necessary conditions to be ordinary/partial differential equations, as opposed to integro-differential equations. It is shown that the index for uniform deposition $\mathcal{V}(\mathbf{p}(\cdot), T)$ does not lend itself to a classical variational form.

The expression for the cost function $\mathcal{V}(\mathbf{p}(\cdot), T)$ in Equation 4 can be written as an integral over time, as opposed to a surface integral. However, it will be shown that the resulting expression is not in a classical variational form because the integrand depends on past time instances (i.e., not just the single value of each time instant). Let a term $\mathcal{V}(\mathbf{p}(\cdot), t)$ be defined as

$$\begin{aligned} \mathcal{V}(\mathbf{p}(\cdot), t) &= \frac{1}{\mathcal{A}_S} \int_{\mathcal{S}} (F(s, \mathbf{p}(\cdot), t) - F_d)^2 ds \\ &= \frac{1}{\mathcal{A}_S} \int_{\mathcal{S}} \left(\int_0^t f(s, \mathbf{p}(\tau)) d\tau - F_d \right)^2 ds. \end{aligned} \quad (7)$$

Differentiating the above expression with respect to t

$$\begin{aligned} \frac{d}{dt} \mathcal{V}(\mathbf{p}(\cdot), t) &= \\ \int_0^t \left(\frac{2}{\mathcal{A}_S} \int_{\mathcal{S}} f(s, \mathbf{p}(t)) f(s, \mathbf{p}(\tau)) ds \right) d\tau \end{aligned}$$

$$-\frac{2}{\mathcal{A}_S} \int_{\mathcal{S}} F_d f(s, \mathbf{p}(t)) ds. \quad (8)$$

Now, $\mathcal{V}(\mathbf{p}(\cdot), T)$ can be written as

$$\begin{aligned} \mathcal{V}(\mathbf{p}(\cdot), T) &= \\ \int_0^T \left\{ \int_0^t \left(\frac{2}{\mathcal{A}_S} \int_{\mathcal{S}} f(s, \mathbf{p}(t)) f(s, \mathbf{p}(\tau)) ds \right) d\tau \right. \\ \left. - \frac{2}{\mathcal{A}_S} \int_{\mathcal{S}} F_d f(s, \mathbf{p}(t)) ds + \frac{F_d^2}{T} \right\} dt. \end{aligned} \quad (9)$$

In contrast, recall that the variation in deposition rate, $\mathcal{D}(\mathbf{p}(\cdot), T)$ is defined as

$$\mathcal{D}(\mathbf{p}(\cdot), T) = \int_0^T \left[\int_{\mathcal{S}} (f(s, \mathbf{p}(t)) - f_d)^2 ds \right] dt. \quad (10)$$

In contrasting the two equations, it is seen that the integrand for the variation in film thickness requires a cumulative evaluation for each point in time, which would yield an integro-differential necessary condition for a minimizing solution. Computing a minimizing solution with this formulation would be computationally expensive and may not yield numerically reliable solutions. Note, however, that the expression for the variation in deposition rate is in the classical form. It is therefore preferable to minimize the variation in deposition rate, as opposed to the variation in film thickness, for selecting a proper spatial path.

Nevertheless, another problem exists with minimizing the variation in deposition rate because the associated necessary conditions are a set of partial differential equations. Therefore, the problem is likely to be ill-posed if no conditioning is placed on the derivative of the path. For example, if at a given section of a surface, it is necessary that a desired deposition rate be achieved, it is likely that, for a symmetric surface and spray distribution, there are at least two points that achieve the same deposition rate. If no conditioning is imposed on the derivative, either of these points can be selected and the resulting solution would not be guaranteed to possess any degree of smoothness.

3.2 Regularization of the Performance Indices

To circumvent this difficulty and to impose a conditioning on the formulation, the performance index is modified to include an arc length term (which effectively constrains the derivative of the path). The physical motivation for this inclusion is as follows: a deposition rate function has a spread that is typically small compared to the actual area that is to be covered. In such a situation, increasing the arc length of the path over the surface increases the coverage

of the surface, thus leading to a greater likelihood of uniform deposition. Therefore, if the performance index is composed of two terms, namely, a deposition/deposition rate uniformization term and a cost associated with the arc length, the relative weights associated with each term defines the solution characteristic to the variational problem. Further, the introduction of the arc length term causes the necessary conditions to be in the form of a set of ordinary differential equations, as opposed to a set of partial differential equations.

Thus, the regularized cost function $\mathcal{R}(\mathbf{p}(\cdot), T)$ is written as the sum of the original cost, $\mathcal{C}(\mathbf{p}(\cdot), T)$, and an arc length term weighted by a regularization parameter λ . Thus

$$\mathcal{R}(\mathbf{p}(\cdot), T) = \mathcal{C}(\mathbf{p}(\cdot), T) + \lambda \int_0^T \|\dot{\mathbf{p}}(t)\|^2 dt. \quad (11)$$

As $\lambda \rightarrow 0$, the new cost function $\mathcal{R}(\mathbf{p}(\cdot), T)$ approaches the deposition/deposition rate based cost function $\mathcal{C}(\mathbf{p}(\cdot), T)$ and as $\lambda \rightarrow \infty$, minimizing $\mathcal{R}(\mathbf{p}(\cdot), T)$ is equivalent to minimizing $\int_0^T \|\dot{\mathbf{p}}(t)\|^2 dt$.

3.3 Necessary Conditions with the Regularized Cost Function

Let the cost function associated with either the variation in film thickness or the variation in deposition rate be denoted by $\mathcal{C}(\mathbf{p}(\cdot), T)$. The regularized version of the uniform deposition problem is stated as

$$\min_{\mathbf{p}(\cdot)} \left\{ \mathcal{C}(\mathbf{p}(\cdot), T) + \lambda \int_0^T \|\dot{\mathbf{p}}(t)\|^2 dt \right\}, \quad (12)$$

where λ is a specified regularization parameter. If the cost function $\mathcal{C}(\mathbf{p}(\cdot), T)$ is written as a time integral in the following form, i.e.,

$$\mathcal{C}(\mathbf{p}(\cdot), T) = \int_0^T \mathcal{K}(\mathbf{p}(t)) dt, \quad (13)$$

then the regularized cost function can be written as

$$\mathcal{R}(\mathbf{p}(\cdot), T) = \int_0^T \left(\mathcal{K}(\mathbf{p}(t)) + \lambda \|\dot{\mathbf{p}}(t)\|^2 \right) dt. \quad (14)$$

The necessary conditions for a minimizing $\mathbf{p}(\cdot)$, minimizing $\mathcal{R}(\mathbf{p}(\cdot), T)$, can be written as

$$\ddot{\mathbf{p}}(t) = \frac{1}{2\lambda} \frac{\partial}{\partial \mathbf{p}} \mathcal{K}(\mathbf{p}(t)). \quad (15)$$

Another feature of the regularization is the fact that the Legendre conditions for a minimizer are satisfied, since

$$\frac{\partial^2}{\partial \mathbf{p}^2} \mathcal{C}(\mathbf{p}(\cdot), T) = 2\lambda > 0. \quad (16)$$

The extension to the case with improved transfer efficiency (i.e., using $\mathcal{C}_a(\mathbf{p}(\cdot), T)$ in place of $\mathcal{C}(\mathbf{p}(\cdot), T)$) is straightforward.

4 Numerical Studies

To demonstrate the utility of the proposed methods, numerical studies are conducted with a simple setup for electrostatic powder coating. For the numerical studies, an exponentially decaying film accumulation function is used, as suggested in [5]. To achieve the objective of uniform deposition, the variation in deposition rate is minimized by solving the necessary conditions to yield minimizing solutions. It is also shown through the numerical studies that minimizing the variation in deposition rate also produces paths that minimize the variation in film thickness.

4.1 Model for Film Accumulation

The type of film accumulation rate function used in this paper, called an infinite range model [7], has the feature that its value actually goes to zero only as the distance between the applicator and the point on the surface tends to infinity. Examples of this type are the bivariate Cauchy distribution considered in [3], and the bivariate Gaussian distribution considered in [6].

An exponential distribution has been proposed in [5] that has been shown to correspond to actual experiments. Thus, a decreasing exponential function will be used for describing the rate of film accumulation at a point on the surface.

4.2 Uniform Coating of a Flat Panel

Numerical studies are conducted by simulating the effect of spraying a flat panel of dimensions $[0, a] \times [0, b]$ on the XY plane. Since the object to be coated is flat, it is assumed that the X axis of the applicator traverses linearly with time. It is assumed that the applicator does not move in the Z direction, and the orientation is assumed normal to the surface. Thus, the single function that is to be determined is the movement along the Y axis, specified by the function $p_y(\cdot)$. For minimizing the variation in deposition rate, the necessary conditions in Equation 15 can be derived as an analytic expression for the assumed film accumulation rate model.

4.3 Effect of Varying the Regularization Parameter

The formulation of the uniform deposition rate problem indicated that, as λ is reduced, more emphasis is placed on minimizing the variation in deposition rate, as opposed to the path length. Correspondingly, in a

numerical study, the solution to the ODE should reflect this behavior. To study the effect of varying λ in a typical situation, numerical studies are conducted on a flat panel within the XY plane, of dimensions $[0, 5] \times [0, 5]$. The values of variation in film thickness and the variation in deposition rates are studied by varying λ . The boundary conditions are fixed at $p_y(0) = 0, p_y(T) = 5$, where T is the total time taken for the spraying process.

Four values of λ are chosen for illustration. The boundary conditions are fixed at $p_y(0) = 0, p_y(5) = 5$, i.e., total painting time is $T = 5$. For each value of λ a relaxation method is employed to solve the two point boundary value problem that yields the minimum deposition rate variation. The behavior of solutions is shown in Figures 1, 2, and 3. An interesting and pos-

λ	Var. in Dep. Rate, $\mathcal{D}(\mathbf{p}(\cdot), T)$	Var. in Film Thickness, $\mathcal{V}(\mathbf{p}(\cdot), T)$
0.1	6.12	0.20
0.01	5.64	0.17
0.001	4.18	0.0211
0.0001	3.94	0.0078

Table 1: Results of varying the regularization parameter λ , with the transfer efficiency parameter $\kappa = 0$.

sibly expected observation is that, the variation in film thickness also reduces with decreasing λ (Table 1).

4.4 Uniform Coating of a Flat Panel with Increased Transfer Efficiency

By varying the regularization parameter, it is seen that the variation in film thickness and deposition rate improve significantly. However, in many cases, it may also be necessary to control the amount of material wasted, i.e., deposited outside the surface. As described in Section 2.2, the transfer efficiency cost term can be included, and the parameter κ can be varied to control the wastage of coating material.

The necessary conditions for the case of the flat panel are now modified to include the contribution of the transfer efficiency term. The necessary conditions can be written in modified form as an analytical equation for the assumed rate of film accumulation.

Suppose the regularization parameter is fixed, the deposition and transfer efficiency can be balanced by adjusting the parameter κ . Numerical simulations are conducted on the flat panel, with the regularization parameter λ fixed at $\lambda = 0.001$. Four non-zero values of κ are used. The results of the studies are shown in Table 2. The resulting spatial paths for one of the cases is shown in Figure 4.

κ	Var. in Dep. Rate, $\mathcal{D}(\mathbf{p}(\cdot), T)$	Var. in Film Thickness, $\mathcal{V}(\mathbf{p}(\cdot), T)$
1.0	6.31	0.26
0.4	6.09	0.19
0.1	5.93	0.0364
0.001	4.70	0.0218

Table 2: Results of varying the transfer efficiency parameter κ . The regularization parameter is fixed at $\lambda = 0.001$.

5 Conclusions

A framework for achieving uniform deposition by selection of a proper spatial path has been presented. It is seen that the variation in deposition rate can be minimized to yield a more uniform coat. A regularization based on arc length is introduced to make the problem well-conditioned. This results in a set of nonlinear ordinary differential equations that are numerically solved to yield optimal paths. A transfer efficiency term is also introduced, along with a transfer efficiency parameter. This parameter can be regulated to control material wastage. A planner based on these studies can also be incorporated in simulation tools for automated coating applications.

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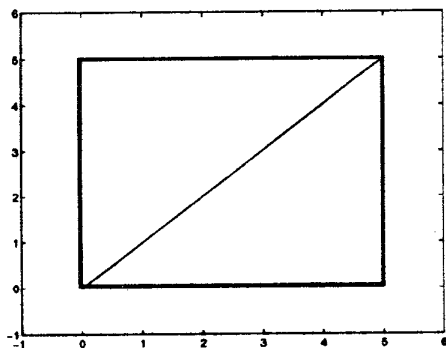


Figure 1: Path for $\lambda = 0.1, \kappa = 0.0$. Emphasis on arc length is high, and hence the path is almost a straight line. The coated plate is over $[0, 5] \times [0, 5]$.

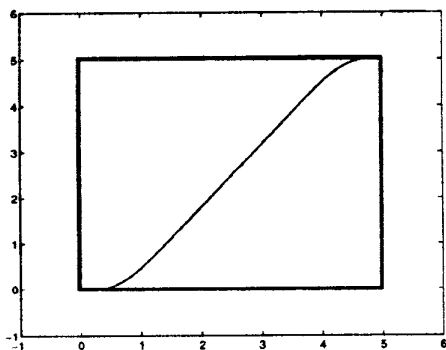


Figure 2: Path for $\lambda = 0.01, \kappa = 0.0$. The emphasis on arc length is reduced, thus causing some curvature near the boundaries.

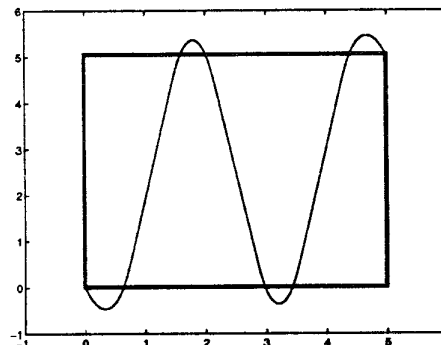


Figure 3: Path for $\lambda = 0.001, \kappa = 0.0$. The effect of increased emphasis on uniform deposition rate is reflected in a spatial path that covers the surface better.

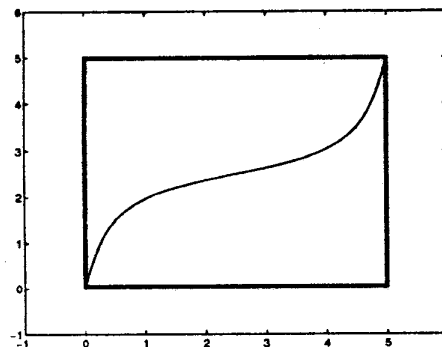


Figure 4: Path for $\lambda = 0.001, \kappa = 1.0$. The spatial path is completely contained within the panel. This causes minimum wastage of coated material, but also increases the variation in film thickness/deposition rate, as compared to the $\kappa = 0.0$ case of Figure 3.