Proceedings of the 30th Conference
on Decision and Control
Brighton, England • December 1991

A Combined Voice and Data Routing Objective †

John K. Antonio
School of Electrical Engineering
Purdue University
West Lafayette, Indiana 47907
(317) 494-6416
jantonio@ecn.purdue.edu

Abstract

A smooth approximation of the Weinstein formula for voice freeze-out fraction is derived. The approximation is a continuously differentiable function of offered voice flow rate. As an important application, it is shown that the approximation can be used as a basis for defining objective functions for optimal routing in voice and voice/data networks. The smoothness of the proposed objective functions suggests the employment of distributed optimal routing algorithms based on nonlinear programming techniques.

I. Introduction

In this paper our focus is on the use of burst switching (BS) as a means of accommodating both digitized voice talkspurts and data messages in a single unified network. Whenever the number of voice packets arriving at a node in a BS network exceeds that node's currently available transmission capacity, then small segments from the head of the excessive voice packets are "frozen" and buffered until a channel becomes available. These frozen voice segments can only be buffered for a relatively short time interval—beyond which additional voice is lost. The congestion of data messages at a node is resolved by simple buffering. Due to the inherently strict time delay constraints associated with voice traffic; it is natural to assume voice has nonpreemptive priority over data at the switching nodes. The excellent paper by Maglaris, *et al.*, [1], includes detailed introductory and background material on modeling of voice and data in BS networks.

II. The Main Result

A. *The Weinstein Formula*

The Weinstein formula [2] for average voice freeze-out fraction Φ is given by

$$\Phi = \frac{1}{Na}\sum_{k=C+1}^{N}(k-C)\binom{N}{k}a^k(1-a)^{N-k}, \qquad (1)$$

where N is the number of sources connected to a channel possessing C subchannels, and a is the so-called activity factor. A typical caller in a conversation produces a short talkspurt approximately 30–40% of the time, thus a reasonable value for a would be between 0.3 and 0.4. With $N = 47$ sources, $C = 24$ channels, and $a = 0.4$, we have $\Phi \approx 0.5\%$, which is the CCITT objective for acceptable voice quality on a TASI link, see reference [1] for further details.

B. *Deriving the Approximation*

Starting with equation (1), we first derive an alternate expression for Φ. It is then shown that this newly derived expression for Φ depends on two values of two separate binomial cumulative distribution functions. The central limit theorem is then applied so as to obtain an approximate expression based on standard normal distributions. Throughout the derivation we implicitly assume that $N > C$. (Whenever $N \le C$, then we obviously have $\Phi = 0$ for any $0 \le a \le 1$.)

$$\begin{aligned}
\Phi &= \frac{1}{Na}\sum_{k=C+1}^{N}(k-C)\binom{N}{k}a^k(1-a)^{N-k} \\
&= \frac{1}{Na}\sum_{k=0}^{N}(k-C)\binom{N}{k}a^k(1-a)^{N-k} \\
&\quad -\frac{1}{Na}\sum_{k=0}^{C}(k-C)\binom{N}{k}a^k(1-a)^{N-k} \\
&= \frac{Na-C}{Na}-\frac{1}{Na}\sum_{k=1}^{C}N\binom{N-1}{k-1}a^k(1-a)^{N-k} \\
&\quad +\frac{C}{Na}\sum_{k=0}^{C}\binom{N}{k}a^k(1-a)^{N-k} \\
&= \frac{Na-C}{Na}-\sum_{l=0}^{C-1}\binom{N-1}{l}a^l(1-a)^{N-1-l} \\
&\quad +\frac{C}{Na}\sum_{k=0}^{C}\binom{N}{k}a^k(1-a)^{N-k}. \qquad (2)
\end{aligned}$$

Note that the two summations in the last expression for Φ are actually the value of two serparate binomial cumulative distribution functions; the first having parameters $N-1$ and a, and evaluated at $C-1$, the second having parameters N and a, and evaluated at C. From the central limit theorem we know that if the random variable X has a binomial distribution with parameters N and $p(=1-q)$; then for any fixed $\alpha < \beta$

$$P(\alpha \le X \le \beta) \to \Gamma\left(\frac{\beta-Np}{\sqrt{Npq}}\right)-\Gamma\left(\frac{\alpha-Np}{\sqrt{Npq}}\right) \quad \text{as } N\to\infty,$$

where $\Gamma(\cdot)$ is the standard cumulative normal distribution. So, our expression for Φ given in equation (2) can be approximated as

$$\begin{aligned}
\tilde{\Phi} &= \frac{V-C}{V}-\Gamma\left(\frac{C-1-V+a}{\sqrt{V(1-a)-a(1-a)}}\right) \\
&\quad +\Gamma\left(\frac{-V+a}{\sqrt{V(1-a)-a(1-a)}}\right) \\
&\quad +\frac{C}{V}\left(\Gamma\left(\frac{C-V}{\sqrt{V(1-a)}}\right)-\Gamma\left(\frac{-V}{\sqrt{V(1-a)}}\right)\right),
\end{aligned}$$

where we have replaced the quantity "Na" with a continuous variable "V," which represents the average offered voice traffic (in Erlangs).

Sample values computed from the actual freeze-out fraction, Φ, and the corresponding values from the approximation, $\tilde{\Phi}$, are included in Table I.[1] For this example we chose $C = 24$ and $a = 0.4$. Note from equation (1) that Φ must be evaluated at the discrete values $N = 0, 1, 2, \ldots$ (which correspond to values of $V = 0.0, 0.4, 0.8, \ldots$). Observe that $\tilde{\Phi} \to \Phi$ for large N, as would be expected from the central limit theorem.

†Supported through Purdue Research Foundation by an XL Summer Faculty Grant.

[1] In computing values from the approximating function, $\tilde{\Phi}$, the integrals associated the standard cumulative normal distributions were evaluated using an IMSL numerical integration subroutine [5].

III. Routing Objective Functions

A. *Voice Only Routing Objective*

As clearly explained in reference [1], the average amount of frozen voice $F(V)$, in Erlangs (as a function of offered voice traffic V) is given by $F(V) = \Phi V$. Therefore, based on our approximation $\tilde{\Phi}$, we note that

$$\tilde{F}(V) = \tilde{\Phi} V$$

is an obvious approximation to the amount of frozen voice. So we have that

$$\begin{aligned}\tilde{F}(V) \stackrel{\text{def}}{=} & (V - C) - V\Gamma\left(\frac{C-1-V+a}{\sqrt{V(1-a)-a(1-a)}}\right) \\ & + V\Gamma\left(\frac{-V+a}{\sqrt{V(1-a)-a(1-a)}}\right) \\ & + C\Gamma\left(\frac{C-V}{\sqrt{V(1-a)}}\right) - C\Gamma\left(\frac{-V}{\sqrt{V(1-a)}}\right). \end{aligned} \quad (3)$$

Sample values of actual and approximated amounts of frozen voice, i.e., $F(V)$ and $\tilde{F}(V)$, are included in Table I; again, we chose $C = 24$ and $a = 0.4$.

By letting $\tilde{F}_{ij}(V_{ij})$ denote the frozen voice (as a function of offered voice V_{ij}) at link (i,j); we note that a natural "voice only" routing objective function is

$$\min\left\{\sum_{i,j} \tilde{F}_{ij}(V_{ij})\right\}.$$

One of the attractive properties of our approximation for voice freeze-out is that it is continuously differentiable. Therefore, the employment of well-established distributed routing algorithms such as [3] and [4] (which are based on nonlinear programming techniques) become a viable approach to solving this (voice only) optimal routing problem.

B. *A Combined Voice and Data Routing Objective*

We now consider the case in which both voice and data are transmitted in the BS network. Our goal is to obtain an estimate of voice freeze-out as a function of both voice and data flow rates, denoted by V and D, respectively. Our approximation is based simply on the requirement that the data traffic is transmitted in finite time. Therefore, if data arrives with rate D, then D subchannels are reserved for data traffic and the remaining $C-D$ channels are used only for voice.

Remarks: (1) We must assume that $D < C$, otherwise average time delay for the data is unbounded. In practice, this assumes some type of flow control is in operation. (2) If the value of $C - D$ is not an integer, then there will be one subchannel that accommodates both data and voice via some form of multiplexing.

Based on the above explanation, it should be clear that a desirable approximation is gotten by replacing all occurrences of "C" in equation (3) with "$C-D$." Therefore, our approximation of voice freeze-out as a function of both voice and data traffic is given by

$$\begin{aligned}\tilde{F}(V,D) \stackrel{\text{def}}{=} & (V - C + D) - V\Gamma\left(\frac{C-D-1-V+a}{\sqrt{V(1-a)-a(1-a)}}\right) \\ & + V\Gamma\left(\frac{-V+a}{\sqrt{V(1-a)-a(1-a)}}\right) + (C-D) \\ & \left(\Gamma\left(\frac{C-D-V}{\sqrt{V(1-a)}}\right) - \Gamma\left(\frac{-V}{\sqrt{V(1-a)}}\right)\right). \end{aligned} \quad (4)$$

Based on the above function, we have the following "voice and data" routing objective:

$$\min\left\{\sum_{i,j} \tilde{F}_{ij}(V_{ij}, D_{ij})\right\}.$$

Similar to the voice only approximation, the approximation of voice freeze-out above is continuously differentiable with respect to both variables V and D. Fig. 1 shows a surface plot of the function $\tilde{F}(V,D)$, with $C = 24$ and $a = 0.4$.

IV. Future Work

Currently, we are investigating the convexity of the approximate formula $\tilde{F}(V)$. Unfortunately, it turns out that $\tilde{F}(V)$ is not convex over the entire interval $V \in [Ca, \infty)$, for all values of C and a. However, it appears that the approximate function is convex over an interval $[(1+\epsilon)Ca, \infty)$, where $\epsilon > 0$ is a small constant which possibly depends on the values of C and a. Due to the fact that $\tilde{F}(V) \approx 0$ for all $V \in [Ca, (1+\epsilon)Ca]$, we do not believe that the non-convexity of the approximation over this small interval will cause any numerical difficulties in determining an optimal solution. For instance, we can simply define the value of the approximate function to be zero for all $V \in [Ca, (1+\epsilon)Ca]$, in order to ensure convexity over the entire interval.

References

[1] B. Maglaris, R. Boorstyn, S. Panwar, T. Spirtos, P. O'Reilly, and C. Jack, "Routing of voice and data in burst-switched networks," *IEEE Trans. on Comm.*, vol. COM-38, pp. 889–897, June 1990.

[2] C. J. Weinstein, "Fractional speech loss and talker activity model for TASI and for packet-switched speech," *IEEE Trans. on Comm.*, vol. COM-26, pp. 1253–1257, Aug. 1978.

[3] R. G. Gallager, "A minimum delay routing algorithm using distributed computation," *IEEE Trans. on Comm.*, vol. COM-25, pp. 73–85, Jan. 1977.

[4] J. N. Tsitsiklis and D. P. Bertsekas, "Distributed asynchronous optimal routing for data networks," *IEEE Trans. Auto. Control* vol. AC-31, pp. 325–332, 1986.

[5] IMSL Library, IMSL Inc., Houston, Texas.

Table I. Comparison of Actual and Estimated Frozen Voice ($C = 24$ and $a = 0.4$)

N	V	Φ	$\tilde{\Phi}$	$F(V)$	$\tilde{F}(V)$
25	10	0.00000	0.00000	0.00000	0.00000
30	12	0.00000	0.00000	0.00000	0.00000
35	14	0.00002	0.00002	0.00022	0.00027
40	16	0.00032	0.00042	0.00513	0.00671
45	18	0.00253	0.00302	0.04560	0.05442
50	20	0.01079	0.01180	0.21573	0.23605
75	30	0.20467	0.20426	6.14015	6.12772
100	40	0.40001	0.40000	16.0005	15.9999

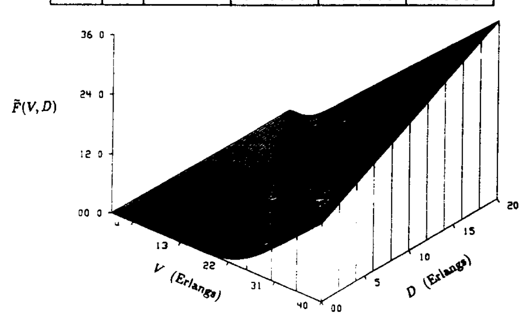

Fig. 1 Plot of $\tilde{F}(V,D)$ with $C = 24$ and $a = 0.4$